A

COMPLETE SYSTEM

OF

MENSURATION

OF

SUPERFICIES AND SOLIDS,

OF ALL REGULAR FIGURES

CALCULATED FOR THE USE OF SCHOOLS, ACADEMIES,
AND PRIVATE LEARNERS.

BY TOBIAS OSTRANDER,
TEACHER OF MATHEMATICS,
And Author of "The Elements of Numbers," "Pla[illegible]
"Mathematical Expositor," &c.

NEW-YORK:
PUBLISHED BY M'ELRATH, BANGS & Co.
W. Osborn & Co., Printers, 85 Chatham-street.

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SECOND EDITION.

NEW YORK:
PUBLISHED BY M'ELRATH, BANGS & CO.
STEREOTYPED BY J. S. REDFIELD.
1833.

Entered according to the Act of Congress, Sept. 1833, by
M‘ELRATH, BANGS & CO.,
In the Clerk's Office of the District Court of the Southern District of New York.

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PREFACE.

The mensuration of superficies and solids, is so important a branch of the mathematical science, that no apology is necessary for presenting this work to the public. It is generally known, that that useful branch of education has been too long neglected, and that no production of the kind, well calculated for the use of common schools, has before this, made its appearance. Those who have written on the subject, have calculated their works more particularly for the use of colleges than other seminaries of learning; and consequently the great mass of the people have not had an opportunity of extending their researches beyond the common limits of plain arithmetic; and the farmer and mechanic, who are immediately concerned, have not had the privilege of becoming acquainted with that branch of education which their daily avocations demand.

The design of the author is, to remove the evil, and present to the public a production necessary to be understood by every class of citizens, and to aid both teachers and students in acquiring a knowledge of this pleasing, useful, and entertaining branch of science.

I have commenced with the work in such a manner, that the student is led along with problems easy and simple, to those of more complicated results, and such as are calculated to exercise his reasoning faculties, and give him an accurate conception of the relation that one figure bears towards another.

In the miscellaneous matter will be found some problems, where the student will have ample scope to exercise his ingenuity and improve his understanding.

The Key is given to aid such teachers as are not sufficiently acquainted with the science to demonstrate the nature and propriety of the theorems, or explain the rules given for the various operations; and also to assist those persons to acquire a knowledge of mensuration, who have not the opportunity of wholly devoting their time to that object.

With the view of aiding the public and being useful to the rising generation, and knowing that every work of a literary nature must either stand or fall by its merit or demerit, I submit the following pages to the consideration of a candid and generous community.

TOBIAS OSTRANDER.

New York, Sept. 1833.

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COMPLETE SYSTEM
OF
MENSURATION.

SECTION I.

The area, or superficial contents of any figure, is the space contained within the line or lines by which the figure is bounded.

OF THE SQUARE.

A square is a figure whose sides are of equal length, and each of the angles contains 90 degrees.

PROBLEM I.

To find the area of a square. *Rule*—Multiply the side by itself; the product will be the area in such terms as correspond with the measure of the sides.

EXAMPLES.

1. What is the area of a square board, each of whose sides are 20 inches in length? Ans. 400 inches.

2. How many square feet in a piece of land ten rods square. Ans. 27225 square feet.

3. How many acres in a square piece of land 60 rods square? Ans. 22,5 acres.

4. How many men may stand on 5 acres of land, each occupying a space of three feet square? Ans. 24,200.

5. How many square links are contained in a piece of land 14 chains long and 14 wide? Ans. 1,960,000.

PROBLEM II.

The area of a square given, to find the length of the side. *Rule*—Extract the square root of the area.

EXAMPLES.

1. The area of a perfect square is 625 feet—What is the length of the side? Ans. 25 feet.

2. What is the length of the sides of a square floor containing 384 square feet? Ans. 19,5959 +.

3. What must be the length of the sides of a square piece of land to contain 6 acres? Ans. 30,9838 + perches.

4. How many chains in length are the sides of a square containing 125 acres of land? Ans. 35,3553 +.

PROBLEM III.

The diagonal* of a perfect square being given, to find the area. *Rule*—Divide the square of the diagonal by 2; the quotient will be the area.

EXAMPLES.

1. The diagonal of a perfect square is 16 chains—How many acres does it contain? Ans. 12,8 acres.

2. The diagonal of a perfect square is 41,3 rods—How many acres does it contain?
Ans. 5 acres, 1 rood, and 13 perches +.

3. The diagonal of a perfect square is 4,78 chains—How many acres does it contain?
Ans. 1 acre and 22 perches +.

PROBLEM IV.

The area of a perfect square being given, to find the length of the diagonal. *Rule*—Extract the square root of double the area.

EXAMPLES.

1. The area of a square piece of land is 64,8 acres—What is the length of the diagonal line in chains?
Ans. 36 chains.

---

* The diagonal is a right line extending from angle to angle.

2. The area of a square piece of land is 578 chains —How many rods is the length of the diagonal line? Ans. 68 rods.

PROBLEM V.

The diagonal of a square being given, to find the side. *Rule*—Extract the square root of half the square of the diagonal, and that will be the length of the side.

EXAMPLES.

1. The diagonal of a square piece of land is 50 chains —What is the length of the side? Ans. 35,3553 + length of the side.

2. The diagonal of a square is 24 rods—What is the length of the side? Ans. 16,97 + rods.

3. The diagonal of a square is 36 feet—What is the length of the side? Ans. 25,4558 +.

PROBLEM VI.

The sides of a square being given, to find the diagonal. *Rule*—Extract the square root of the sum of the squares of the two adjacent sides.

EXAMPLES.

1. The sides of a perfect square are each 19 chains —What is the length of the diagonal line? Ans. 26,87 + chains.

2. The sides of a perfect square are each 24 rods—What is the length of the diagonal line? Ans. 33,9411 + rods.

3. The sides of a square are each 43 feet—What is the length of the diagonal line? Ans. 60,8111 feet.

PROBLEM VII.

The difference between the diagonal of a square and the length of the side being given, to find the area. *Rule*—Extract the square root of twice the square of the given difference, and to that root add the given difference; the sum will be the length of the side. Square the length of the side, and that will be the area required.

EXAMPLES.

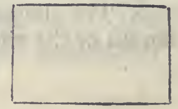
1. The difference between the diagonal of a square and the length of the side is 20 chains—What is the area? Ans. 233 acres, 0 roods, 21,92 rods.

2. The diagonal of a square is 12 rods longer than the side—How many acres does it contain?
Ans. 5 acres, 0 roods, 39 rods.

3. The diagonal of a square is 8 feet longer than the side—What is the contents of the square?
Ans. 373 + feet.

OF THE RECTANGLE.

A rectangle is a parallelogram whose opposite sides are parallel, and whose angles are all right angles containing 90 degrees each.



PROBLEM I.

The length and breadth of a rectangular parallelogram are given, to find the area. *Rule*—Multiply the length by the breadth; the product will be the area.

EXAMPLES.

1. The longest side of a rectangular parallelogram is 24 rods, and the shortest 16—What is its content?
Ans. 384 rods.

2. How many acres in a parallelogram whose sides are 36 and 18 chains? Ans. 64,8 acres.

3. How many square feet in a board 14 feet long and 15 inches wide? Ans. 17,5 feet.

4. How many square feet in 20 boards, each 16 feet long, and 19 inches wide? Ans. 506$\frac{2}{3}$ feet.

5. How many squares of 100 feet each are there in a floor 36 feet long and 18 feet wide? Ans. 6,48 squares.

PROBLEM II.

The area and either side of a rectangular parallelogram are given, to find the shortest side. *Rule*—Divide the area by the given side, the quotient will be the other.

EXAMPLES.

1. The area of a rectangular parallelogram is 456 chains, and the longest side is 30 chains—What is the length of the shortest? Ans. 15,20 chains.

2. The area of a rectangular parallelogram is 784 feet, and the shortest side is 12 feet—What is the longest side? Ans. 65,333 feet, or $65\frac{1}{3}$ feet.

3. The area of a rectangular parallelogram is 36 acres, 3 roods, and 30 perches, and the longest side is 120 rods—What is the length of the shortest?

Ans. 49,25 rods length of the shorter side.

PROBLEM III.

The area of a rectangular parallelogram is given and the sum of the longest and shortest sides, to find the sides. *Rule*—From the square of half the sum subtract the area, and the square root of the remainder added to the half sum will give the longest side; subtracted leaves the shortest.

EXAMPLES.

1. The area of a rectangular parallelogram is 576 chains, and the sum of the longest and shortest sides is 60 chains—What is the length and breadth of the parallelogram? Ans. 48 chains long, and 12 in breadth.

2. A gentleman has a garden in the form of a rectangular parallelogram containing 3 acres, 1 rood, and 32 rods, the wall that surrounds it is 116 rods in length—What is the length and breadth of the garden?

Ans. 46 and 12 rods.

3. A gentleman owns a garden in the form of a rectangular parallelogram 32 rods long and 8 wide, and is desirous of making a gravel walk half round which shall take up $\frac{1}{8}$ of the ground—What must be the width of the walk? Ans. ,7335 of a rod.

PROBLEM IV.

The area of a rectangular parallelogram is given and the difference of the sides, to find the sides. *Rule*—To the area add the square of half the difference of the two sides, and the square root of the sum will be the half sum of the two sides; to the half sum add half the difference and the sum will be the longest side; subtracted leaves the less.

EXAMPLES.

1. The area of a rectangular parallelogram is 4 acres, and the longest side exceeds the shortest by 12 rods—What are the length of the sides?

Ans. The longer side is 32 rods, the shorter 20.

2. The longest side of a rectangular parallelogram exceeds the shortest by 16 chains, and the area is 138 acres—What is the length of the sides?

Ans. 46 and 30 chains.

PROBLEM V.

The area of a rectangular parallelogram is given, and the proportion of the two sides, to find the sides. *Rule*—Multiply the area by the less number of the proportion, and divide the product by the greater, the square root of the quotient will be the length of the shortest side; the contrary will give the length of the longest.

EXAMPLES.

1. The area of a certain piece of land is 40 acres, 3 roods, and 24 rods, and the length is to the breadth as 3 to 4, and the form a rectangular parallelogram—What is the length of the sides?

Ans. 70,057 and 93,4093 rods.

2. The area of a rectangular parallelogram is 24 acres, and the length is to the breadth as 2 to 3.—What is the length of the sides? Ans. 12,6491 chains the shorter side, and 19,02365 longer side.

PROBLEM VI.

The difference of the two sides of a rectangular parallelogram is given, and the difference of their squares to find the sides. *Rule*—Divide the difference of their squares by the difference of the sides, and the quotient will be their sum; then to half their sum add half their difference, and the sum will be the longest side—subtracted leaves the less.

EXAMPLES.

1. The difference of the squares of the two adjacent sides of a rectangular parallelogram is 720 rods, and the difference of those sides 12 rods—Required the length of the sides of the parallelogram.

Ans. 36 rods the longest side, and 24 rods the shorter.

2. The difference of the squares of the two adjacent sides of a rectangular parallelogram is 108 chains, and the difference of those sides 6 chains—Required the sides of the parallelogram.

Ans. 12 chains the longer, and 6 chains the shorter.

PROBLEM VII.

When the sum of the two adjacent sides of a rectangular parallelogram is given, and the difference of their squares, to find the length of the sides. *Rule*—Divide the difference of their squares by their sum, and the quotient will be the difference of the two sides: to half the given sum add half the difference, and the sum will be the length of the longest side—subtracted leaves the shortest.

EXAMPLES.

1. The sum of the two sides of a rectangular parallelogram is 20 chains, and the difference of the squares of the two adjacent sides is 80 chains—Required the length of the sides.

Ans. The sides are 12 and 8 chains.

2. The sum of the two adjacent sides of a rectangular parallelogram is 16 feet, and the difference of the squares of those sides is 32 feet—Required the sides of the parallelogram. Ans. 9 and 7 feet.

PROBLEM VIII.

Having the difference of the two adjacent sides of a rectangular parallelogram, and the sum of their squares given, to find the sides. *Rule*—From the sum of their squares take the square of their difference, add the remainder to the sum of their squares, and the square root of this last sum will be the sum of the two numbers, then to half the sum add half the given difference and the sum will be the longest side, subtracted leaves the less.

EXAMPLES.

1. The difference between the two adjacent sides of a rectangular parallelogram is 8 chains, and the sum of the squares of those sides is 194—What is the length and breadth of the parallelogram?

Ans. 13 and 5 chains.

2. The difference between the two adjacent sides of a rectangular parallelogram is 11 inches, and the sum of the squares of those sides is 281—What is the length and breadth of the parallelogram?

Ans. 16 and 5 inches.

PROBLEM IX.

Having the sum of the two adjacent sides of a rectangular parallelogram, and the sum of the squares of those sides given, to find the area. *Rule*—From the square of their sum take the sum of their squares, then from the sum of their squares take this remainder, and the square root of the difference will be the difference of the two numbers; to half their sum add half their difference and the sum will be the length of the longest side, subtracted leaves the shortest, then multiply the length by the breadth and the product will be the area.

EXAMPLES.

1. The sum of the two adjacent sides of a rectangular parallelogram is 50 chains, and the sum of the squares of those sides is 1300—Required the area of the paral-

lelogram. Ans. 30 and 20 chains, the length of the sides required, and the area, 60 acres.

2. The sum of the squares of the two adjacent sides of a gentleman's garden, in the form of an oblong square or rectangular parallelogram, is 464 rods, and the length of the wall that encloses it, is 56 rods—Required the area of the parallelogram. Ans. 1 acre.

PROBLEM X.

Having the area of a parallelogram, and the length of the diagonal line given, to find the length of the sides. *Rule*—From the square of the diagonal, subtract twice the area, and the square root of the remainder will be the difference of the two adjacent sides. To the square of the diagonal add twice the area, and the square root of the sum will be the sum of the two sides; then to half the sum add half the difference, and the sum will be the length of the longest side—subtracted, leaves the shortest.

EXAMPLES.

1. The area of a rectangular parallelogram is 30 acres, and the length of the diagonal is 25 chains—Required the sides of the parallelogram.

Ans. 20 and 15 chains the length and breadth.

2. The area of a rectangular parallelogram is 6 acres and 12 rods, and the length of the diagonal is 45 rods—What is the length of the sides of the parallelogram? Ans. 36 rods the length, and 27 the breadth.

PROBLEM XI.

Having the area of a rectangular parallelogram, and the difference of the squares of the two adjacent sides given, to determine the length of the sides of the parallelogram. *Rule*—To the square of the area add the square of half the difference of their squares, and, from the square root of that sum, subtract half the difference of their squares, and the remainder will be the square of the shortest side; extract the square root and that will be the length of the shortest side—divide the area by the shortest side, and the quotient will be the longest.

EXAMPLES.

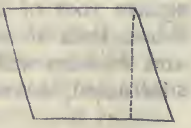
1. The area of a rectangular parallelogram is 4 acres, 3 roods, and 8 rods, and the difference of the squares of the adjacent sides is 28 chains—What is the length of the sides? Ans. 8 chains by 6.

2. The difference of the squares of the adjacent sides of a parallelogram is 128 chains, and the area 48 chains—What is the length of the sides?

Ans. 12 chains by 4.

OF THE RHOMBUS.

A rhombus is a quadrilateral figure, whose sides are equal, but whose angles are not right angles.



PROBLEM I.

To find the area of a rhombus.

Rule—Multiply the length of one side by a perpendicular let fall from one of the angles to the opposite side.

EXAMPLES.

1. The length of the sides of a rhombus are each 12,24 feet, and the perpendicular height 9,16 feet—Required its area. Ans. 112,1184 feet.

2. Required the area of a rhombus, whose length is 12 feet, 6 inches, and its height 9 feet, 3 inches.

Ans. 115,625 feet.

PROBLEM II.

The area of a rhombus, and the length of the side being given, to find the perpendicular height. *Rule*—Divide the area by the length of the side, the quotient will be the length of the perpendicular; or, divide by the height, the quotient will be the length of the side.

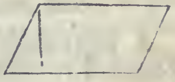
EXAMPLES.

1. The area of a rhombus is 24 rods, and the length

of the side 5 rods—What is the length of the perpendicular? Ans. 4,8 rods the perpendicular.

2. The area of a rhombus is 125 feet, and the perpendicular height, 8 feet, 6 inches; required the length of the side? Ans. 14,7 + feet.

OF THE RHOMBOID.



A rhomboid, or rhomboides, is a quadrilateral figure, whose sides are parallel, and angles not right angles.

PROBLEM I.

To find the area of a rhomboid. *Rule*—Multiply the length by a perpendicular let fall from one of the angles to the opposite side.

EXAMPLES.

1. The length of a rhomboid is 26 rods, and the perpendicular height 8 rods—Required the area.
Ans. 1 acre, 1 rood, and 8 rods.

2. The length of a rhomboid is 18 feet, 9 inches, and the perpendicular height 12 feet, 3 inches—How many square yards does it contain?
Ans. 25,5208 + square yards.

3. The length of a rhomboid is 36 feet, and the perpendicular height 24 feet, 9 inches—How many square rods does it contain? Ans. $3\frac{3}{11}$ square rods.

PROBLEM II.

When the area of a rhomboid, and the length are given, to find the perpendicular. *Rule*—divide the area by the length, the quotient will be the perpendicular; or, divide the area by the perpendicular, the quotient will be the length.

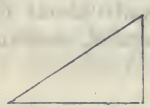
EXAMPLES.

1. The area of a rhomboid is 4 acres, 3 roods, and 12 perches, and the length 9 chains and 20 links—What is the length of the perpendicular?
Ans. 5,2445 + chains.

2. The area of a rhomboid is 47 feet 9 inches, and the perpendicular is 4 feet, 8 inches—What is the length? Ans. 12,141 + feet the length.

3. The area of a rhomboid is one square rod, and the perpendicular is 7 feet, 2 inches—What is its length?
Ans. 37,99—feet.

OF RIGHT-ANGLED TRIANGLES.



A right-angled triangle, is a figure bounded by three sides, and has one right angle: the longest side is called the hypotenuse, and the other two, the legs, otherwise known by the names of base, and perpendicular.

PROBLEM I.

To find the area of a right-angled triangle, the base and perpendicular being given. *Rule*—Multiply half the length of the base by the length of the perpendicular, the product will be the area; or half the perpendicular by the whole length of the base, the product will be the area.

EXAMPLES.

1. The base of a right-angled triangle is 36 rods, and the perpendicular 12 rods—How many acres does it contain? Ans. 1 acre, 1 rood, and 16 rods.

2. The perpendicular of a right-angled triangle is 24 chains and 76 links, and the base 41 chains and 23 links—How many acres does it contain?
Ans. 51 acres, 6$\frac{8}{10}$ rods.

PROBLEM II.

When the area and the length of either base or perpendicular are given, to find the other. *Rule*—divide twice the area by the base, the quotient will be the perpendicular; or, divide by the perpendicular, the quotient will be the base.

EXAMPLES.

1. The area of a triangle is 16 feet 6 inches, and the

base is 7 feet 2 inches—Required the perpendicular.
Ans. 4,604 + feet.

2. The area of a triangle is 4 acres, 1 rood, and 20 rods, and the perpendicular is 3 chains and 14 links—What is the length of the base? Ans. 27,8662 chains.

PROBLEM III.

When the base and perpendicular are given, to find the hypotenuse. *Rule*—Extract the square root of the sum of the squares of the base and perpendicular, and that will be the length of the hypotenuse.

EXAMPLES.

1. The base of a right-angled triangle is 16 chains, and the perpendicular 12 chains—Required the length of the hypotenuse. Ans. 20 chains.

2. The perpendicular of a right-angled triangle is 4 feet 6 inches, and the base 7 feet 9 inches—Required the length of the hypotenuse. Ans. 8,9617 + feet.

PROBLEM IV.

When the hypotenuse and either leg of a right-angled triangle are given, to find the other. *Rule*—Extract the square root of the difference of the squares of the hypotenuse and given leg, and that will be the length of the other.

EXAMPLES.

1. The hypotenuse of a right-angled triangle is 25 chains, and the base 20 chains—What is the length of the perpendicular. Ans. 15 chains.

2. The hypotenuse of a right-angled triangle is 40 chains, and the perpendicular 26 chains—What is the length of the base? Ans. 30,38 chains.

3. A ladder 30 feet long, placed near the middle of a street, reached the buildings at one side 24 feet from the ground; and the opposite side, without moving the foot, 18 feet—What was the breadth of the street?
Ans. 42 feet.

4. A line, reaching from the top of a fort built on the top of a rock, is 120 yards from the top of the rock—to the place of observation 100 yards—and from that part of the rock level with the eye 80 yards—The height of the fort is required. Ans. 29,4427 + yards.

PROBLEM V.

The base of a right-angled triangle and the difference between the hypotenuse and perpendicular are given, to find the perpendicular. *Rule*—From the square of the base subtract the square of the difference; divide the remainder by twice the difference, and the quotient will be the perpendicular.

EXAMPLES.

1. The base of a right-angled triangle is 20 chains, and the difference between the hypotenuse and perpendicular is 2 chains—What is the length of the perpendicular? Ans. 99 chains.

2. The hypotenuse of a right-angled triangle is 5 chains longer than the perpendicular; the base is 24 chains—What is the area of the triangle?

Ans. 66,12 acres.

3. The hypotenuse of a right-angled triangle is 6,14 feet longer than the perpendicular—Required the perpendicular, when the base is 24 feet.

Ans. 43,8355 feet.

PROBLEM VI.

The base of a right-angled triangle and the difference between the perpendicular and hypotenuse are given, to find the hypotenuse. *Rule*—To the square of the base add the square of the difference; divide the sum by twice the difference, and the quotient will be the hypotenuse.

EXAMPLES.

1. The hypotenuse of a right-angled triangle is 4 feet longer than the perpendicular; the base is 40 feet—What is the length of the hypotenuse? Ans. 202 feet.

2. What is the length of the hypotenuse of a right-angled triangle, if it be 7 feet longer than the perpendicular, when the base is 30 feet? Ans. 67,8 —.

PROBLEM VII.

The base of a right-angled triangle and the sum of the hypotenuse and perpendicular are given, to find the perpendicular. *Rule*—From the square of the sum subtract the square of the base, divide the remainder by twice the sum, and the quotient will be the perpendicular.

EXAMPLES.

1. The sum of the hypotenuse and perpendicular are 100 feet, and the length of the base 40 feet—What is the length of the perpendicular? Ans. 42 feet.

2. The height of a tree, standing perpendicularly on a plane, is 120 feet—At what height must it break off so that the top may rest on the ground 40 feet from the base, and the place broken on the upright part?
Ans. 53 feet 4 inches.

3. The hypotenuse of a right-angled triangle is 30 chains; and if a perpendicular be erected from the end of the hypotenuse to intersect a line drawn from the point of the triangle, the length of that line and perpendicular will be 50 chains—What is the length of the perpendicular? Ans. 16 chains.

PROBLEM VIII.

The sum of the hypotenuse and perpendicular and the length of the base of a right-angled triangle are given, to find the hypotenuse. *Rule*—To the square of the sum add the square of the base, and divide the number by the sum of the hypotenuse and perpendicular, and the quotient will be the length of the hypotenuse.

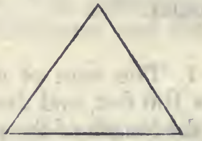
EXAMPLES.

1. The base of a right-angled triangle is 24 chains,

and the sum of the hypotenuse and perpendicular is 60 chains—What is the length of the hypotenuse?
Ans. 34,8 chains.

2. A tree 100 feet high, growing perpendicular on a plane, was broken off by a blast of wind; the broken part resting on the upright, and the top on the ground 30 feet from the base—What was the length of the broken part? Ans. 54,5 feet.

OF EQUILATERAL TRIANGLES.



An equilateral triangle is one whose sides are of equal length, and consequently the angles are all equal, each containing 60 degrees.

PROBLEM I.

To find the area of an equilateral triangle. *Rule*—multiply the square of the side by ,433013, and the product will be the area.

EXAMPLES.

1. What is the area of an equilateral triangle whose side is 20 chains? Ans. 17,32052 acres.
2. How many square yards in an equilateral triangle whose side is 30 feet? Ans. 43,3013 yards.
3. How many acres in an equilateral triangle whose side is 40 rods? Ans. 4,33005 acres.

PROBLEM II.

The side of an equilateral triangle given, to find the perpendicular. *Rule*—From the square of the given side subtract the square of half the side, and the square root of the remainder will be the perpendicular.

EXAMPLES.

1. What is the length of a perpendicular dropped from one of the angles to the middle of its opposite side of an equilateral triangle, when the side is 20 chains?
Ans. 17,32 chains.

2. What is the length of a perpendicular let fall from any angle of an equilateral triangle to its opposite side, when the length of the side is 12 chains?

Ans. 10,3923 + chains.

PROBLEM III.

The area of an equilateral triangle and the perpendicular given, to find the side. *Rule*—Divide twice the area by the perpendicular, and the quotient will be the length of the side.

EXAMPLES.

1. The area of an equilateral triangle is 11 acres, 3 roods, and 16 rods, and the perpendicular 13,8564 chains —What is the length of the side? Ans. 68,416 + rods.

2. The area of an equilateral triangle is one acre, 2 roods, and 15 rods, and the perpendicular is 21,26 rods —What is the length of the side? Ans. 23,9887 + rods.

PROBLEM IV.

The area and side of an equilateral triangle given, to find the perpendicular. *Rule*—Divide double the area by the given side, and the quotient will be the perpendicular.

EXAMPLES.

1. The length of the side of an equilateral triangle is 16 chains, and the area 11 acres, 3 roods, and 16 rods —What is the length of the perpendicular?

Ans. 14,8125 chains.

PROBLEM V.

The area of an equilateral triangle being given to find the side. *Rule*—Divide the area by the decimal ,433013, and extract the square root of the quotient.

EXAMPLES.

1. The area of an equilateral triangle is 24 acres— What is the length of the sides? Ans. 23,5426 chains.

2. The area of an equilateral triangle is 4 acres, 2

roods, and 5 rods—How many chains in the length of the side. Ans. 10,229 chains.

3. The area of an equilateral triangle is one square chain—How many feet in the length of the side.
Ans. 100,29 feet.

PROBLEM VI.

When the length of three perpendiculars let fall from a point within an equilateral triangle to the opposite sides, are given, to determine the length of the sides. *Rule*—Add the length of the three perpendiculars together, and divide the sum by half the square root of the number three, the quotient will be the length of the side.

EXAMPLES.

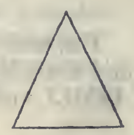
1. The sum of three perpendiculars let fall from a point within an equilateral triangle is 45 chains and 40 links—What is the length of the sides of the triangle.
Ans. 52,4233 chains.

2. The three perpendiculars let fall from a point within an equilateral triangle are 26,32 and 19 rods—How many acres are contained in the triangle?
Ans. 21,39 — acres.

3. The three perpendiculars let fall from a point within an equilateral triangle are 16 feet 6 inches, 24 feet, and 32 feet six inches—How many square rods does the triangle contain. Ans. 1,13 square rods.

OF THE ISOSCELES TRIANGLE.

An isosceles triangle is that which has two sides of equal length, consequently two of the angles are also equal.



PROBLEM I.

To find the area of an isosceles triangle having the length of the sides given. *Rule*—From the square of one of the equal sides subtract the square of half the unequal side, and the square root of the remainder will be the length of a perpendicular let fall from the vertical angle to the centre of the base or unequal side; multiply half the length of the base by the perpendicular, and the product will be the area.

EXAMPLES.

1. The two equal sides of an isosceles triangle are each 24 feet, and the base 32 feet—How many square rods does the triangle contain?
Ans. 1,0513 — square rods.

2. The base of an isosceles triangle is 25 chains, and each of the equal sides 30 chains—How many acres are contained in the triangle? Ans. 34 acres and 14 rods.

PROBLEM II.

The area of an isosceles triangle and the length of the base being given, to find the length of each of the equal sides. *Rule.*—Divide double the area by the base, and the quotient will be the perpendicular; then to the square of the perpendicular add the square of half the unequal side, and the square root of the sum will be the length of each of the equal sides.

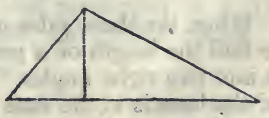
EXAMPLES.

1. The area of an isosceles triangle is 4 square rods, and the base 18 feet—What is the length of each of the equal sides? Ans. 121,3342 feet.

2. The area of an isosceles triangle is 48 rods, and the length of the base 8 rods—What is the length of each of the equal sides? Ans. 12,649 + rods.

OF THE SCALENE TRIANGLE.

A scalene triangle is that whose sides are of different lengths, and consequently, the angles are unequal.



PROBLEM I.

To find the area of a scalene triangle, the base and perpendicular being given. *Rule*—Multiply the base by half the perpendicular: the product will be the area.

EXAMPLES.

1. The base of a scalene triangle is 18 rods, and the perpendicular 9 rods—How many square chains does the triangle contain? Ans. $5\frac{1}{16}$ square chains.

2. The base of a scalene triangle is 15 chains, and the perpendicular 18 feet—How many square rods does the triangle contain? Ans. 32,7272 square rods.

3. The base of a scalene triangle is 18 rods, and the area 4 acres—What is the length of the perpendicular?
Ans. $71\frac{2}{9}$ rods.

PROBLEM II.

When the three sides of a scalene triangle are given, to find the area. *Rule*—Add the three sides together, and from their half sum subtract each side separately; multiply the half sum and their remainders together, and the square root of the continual product will be the area.

EXAMPLES.

1. The sides of a scalene triangle are 16, 18, 24 chains—How many acres does it contain?
Ans. 14,4 — acres.

2. The three sides of a scalene triangle are 12, 16, 20 rods—How many square rods does it contain?
Ans. 96 square rods.

3. How many square rods in a scalene triangle whose sides are 24, 36, and 44 feet? Ans. 2,46 — nearly.

PROBLEM III.

When the three sides of a scalene triangle are given, to find the length of a perpendicular which will divide it into two right angles. *Rule*—As the whole length of the base is to the sum of the other two sides, so is the difference of those sides to the difference of the segments of the base; half the said difference added to half the base, gives the longest of the two segments—subtracted, leaves the less; then from the square of the shortest side subtract the square of the shortest segment, and the square root of the remainder will be the length of the perpendicular.

EXAMPLES.

1. The three sides of a scalene triangle is 14, 18, and 20 feet—What is the length of a perpendicular which will divide it into two right-angled triangles? Ans. 12,2376 + feet.

2. The three sides of a scalene triangle are 40, 60, 80 chains—What is the length of a perpendicular which divides it into two right angles. Ans. 29,0473 + chains.

PROBLEM IV.

The proportion of the three sides of a scalene triangle are given, to find the length of the sides of a triangle corresponding with a given area. *Rule*—Find the area of the triangle according to the given proportion; then as that area is to the square of either of its sides, so is the area given to the square of its similar side.

EXAMPLES.

1. The three sides of a triangle are in the proportion of 3, 4, and 6 chains—Required the length of the sides of a triangle in the same proportion, containing one acre. Ans. 4,1082, 5,4776, and 8,2164 chains.

2. The area of a piece of land is 120 acres, 3 roods, and 4 rods, in the form of a scalene triangle—What must be the length of the sides in the proportion of 9, 8, and 6? Ans. 172, $229\frac{2}{3}$, and 258 rods.

3. What is the length of the sides of a scalene triangle containing 10 acres, in the proportion of 5, 11, 13? Ans. 9,64, 21,208, and 25,064 chains.

PROBLEM V.

The area one side and the sum of the three sides of a scalene triangle being given, to find the other two sides. *Rule*—Multiply the square of the half sum by the square of the difference between the half sum and the sum of the unknown sides, and to their product add the square of the area for a dividend. Multiply the half sum by the said difference for a divisor, the quotient will be the product of the two unknown sides; then from the square of the half sum of the two unknown sides subtract their product, and the square root of the remainder will be the half difference; which, added to the half sum, will give the longest of the unknown sides—subtracted, will leave the shortest.

EXAMPLES.

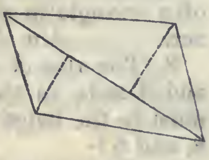
1. The sum of the three sides of a scalene triangle is 40 chains; the one is 18, and the area is 5 acres, 2 roods, and 25 rods—What is the length of each of the other sides? Ans. 48,032, and 39,968 rods.

2. One side of a scalene triangle is 20 rods, and the sum of the other two is 50—What must the length of each be so that the triangle shall contain one acre? Ans. 32,158 and 17,842 rods.

3. One side of a scalene triangle is 20 feet, and the sum of the other two is 30; the square of the area is 11375—What is the length of each of the other sides? Ans. 18 and 12 feet.

OF THE TRAPEZIUM.

A trapezium is an irregular quadrilateral figure, whose sides are of different lengths, and consequently all the angles different.



PROBLEM I.

To find the area of a trapezium, the length of the diagonal and perpendiculars being given. *Rule*—Multiply the sum of the perpendiculars by the diagonal, and half the product will be the area.

EXAMPLES.

1. What is the area of a trapezium whose diagonal

is 24 chains and 27 links, the sum of the two perpendiculars let fall from the angles to the diagonal, being 21 chains, and 14 links?

Ans. 25 acres, 2 roods, and 24 rods.

2. How many acres in a trapezium whose diagonal is 33 rods, and the sum of the perpendicular 24 rods?

Ans. 2 acres, 1 rood, and 36 rods.

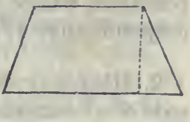
3. How many square rods in a trapezium whose diagonal is 108 feet 6 inches, and the perpendiculars 56 feet 3 inches, and 60 feet, 9 inches?

Ans. 23,314 square rods.

4. How many acres in a trapezium whose diagonal is 80,5 chains, and the perpendiculars 22,4, and 28,3 chains? Ans. 204 acres, 2 roods, and 28 rods.

OF THE TRAPEZOID.

A trapezoid is a quadrilateral figure having two parallel sides.



PROBLEM I.

To find the area of a trapezoid having the length of the two parallel sides and the perpendicular distance between them given. *Rule*—Add the two parallel sides together, and multiply the sum by the perpendicular, half the product will be the area.

EXAMPLES.

1. The parallel sides of a trapezoid are 24 chains and 46 links, and 38 chains and 40 links, and the perpendicular 16 chains and 20 links—How many acres does it contain? Ans. 50 acres, 3 roods, and 26 rods.

2. The perpendicular distance between the two parallel sides of a trapezoid is 4,5 rods, and the length of the parallel sides are 12,75 rods, and 16,67 rods—How many square rods does it contain?

Ans. 66,195 square rods.

3. The length of the parallel sides are 14 feet 6 inches, and 24 feet 9 inches, and the perpendicular dis-

tance between them is 8 feet 3 inches—How many square feet does it contain. Ans. 161,9 + feet.

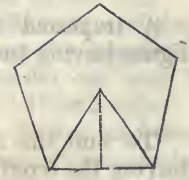
4. How many square rods are contained in a trapezoid whose parallel sides are 5 chains 16 links, and 9 chains 14 links, and their perpendicular distance 3 chains 7 links? Ans. 351,2 square rods.

OF POLYGONS.

PROBLEM I.

To find the area of any regular polygon. *Rule.*—Multiply half the perimeter* by a perpendicular falling from the centre of the figure upon the middle of one of the sides, and the product will be the area of the polygon.

EXAMPLES.



1. Required the area of a regular pentagon whose side is 25 feet, and perpendicular 17,2 feet.
Ans. 1075 feet.

2. Required the area in square rods of a hexagon, whose side is 14,6 feet, and perpendicular 12,64.
Ans. 2,0335 square rods.

3. How many acres are contained in a regular heptagon, whose sides are each 19,38 chains, and perpendicular from the centre 20 chains? Ans. 135,66 acres.

4. How many square rods are contained in an octagon, whose sides are each 9,941 feet, and the perpendicular from the centre 12 feet?
Ans. 1,7526 square rods.

PROBLEM II.

To find the area of any regular polygon when the side only is given. *Rule.*—Multiply the square of the side of the polygon by the number standing opposite to its name in the following table, and the product will be the area.

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* The Perimeter of any figure is the sum of all its sides.

No. of sides.		Areas of the following regular figures whose sides are one.	Radius of circumscribing circles.
3	Trigon, or triangle	0,4330127	0,5773503
4	Tetragon, or square	1,0000000	0,7071068
5	Pentagon	1,7204774	0,8506508
6	Hexagon	2,5980762	1,0000000
7	Heptagon	3,6339124	1,1523824
8	Octagon	4,8284271	1,3065628
9	Nonagon	6,1818242	1,4619022
10	Decagon	7,6942088	1,6180340
11	Undecagon	9,3656399	1,7747324
12	Dodecagon	11,1961524	1,9318517

The above table should be made by all students who understand plain Trigonometry.

Note.—The multipliers named in the table are the areas of their figures whose side is one.

EXAMPLES.

1. The side of a pentagon is 25 feet—What is its area?
Ans. 1075,298 + feet.

2. The side of a hexagon is 24 feet—What is its area? Ans. 1496,49 + feet.

3. What is the area of a heptagon whose side is 16 feet? Ans. 930,28 + square feet.

4. How many square yards in an octagon whose side is 12 feet 6 inches? Ans. 83,8268 + square yards.

5. How many acres in an enneagon whose sides are each 15 chains in length?
Ans. 139 acres and 14 perches.

6. How many pieces each 4 inches square may be cut from a decagon whose side is 12 inches?
Ans. 69,248 — pieces.

7. How many square rods are contained in an undecagon whose side is 4 rods in length?
Ans. 149,85 + square rods.

8. Each side of a dodecagon is 9 inches—How many square feet does it contain? Ans. 62,98 —.

PROBLEM III.

When the area of any regular polygon is given, to find the side. Divide the area by the number in the table corresponding with the figure, and the square root of the quotient will be the length of the side.

EXAMPLES.

1. The area of a pentagon is 4 acres—How many perches are contained in the side?

Ans. 19,287 + perches

2. The area of an octagonal floor is 560 feet—What is the length of the side? Ans. 10,769 + feet.

3. Required the length of the side of a hexagon containing one acre.

Ans. 7,8475 + perches the length of the side.

TO DIVIDE A SQUARE.

PROBLEM I.

To cut off a given area from a square, parallel to either side. *Rule*—Divide the given area by the length of the side; the quotient will be the length on the other side.

EXAMPLES.

1. The sides of a perfect square are 16 chains—What must be the length of the other side to leave 15 acres at the end? Ans. 9,375 chains.

2. What length must be cut off from a square whose sides are 20 chains and 14 links, to leave 10 acres at the end? Ans. 4,985.

3. The area of a square piece of land is 5 acres, 2 roods, and 14 rods—What is the length of that piece, the breadth remaining the same, which shall contain 3 acres and 6 rods of land?

Ans. 16,254 rods the length of the side.

PROBLEM II.

To cut off a given quantity from the angle of a square, the length of the sides being given. *Rule*—Find the area of half the given square; then say, as half the given area is to the square of the given side, so is the area left at the verticle angle to the square of its side—the square root of which will be the length on each side.

EXAMPLES.

1. The length of the side of a perfect square is 10 chains and 18 links—What is the length of the sides of an isosceles triangle, cut from the angle of this square, containing 4 acres? Ans. 8,9442 +.

2. The area of a square is one acre—What length shall I run from either corner to cut off an isosceles triangle containing twenty-five rods? Ans. 7,071 rods.

TO DIVIDE A PARALLELOGRAM.

PROBLEM I.

The sides of a parallelogram are given, to cut off a given area parallel to either side. *Rule*—Divide the area given by the side which is to retain its length, and the quotient will be the length of the other side.

EXAMPLES.

1. The sides of a rectangular parallelogram are 18 chains and 16 links, and 12 chains 15 links—What must the length be to leave 12 acres adjoining the shortest side? Ans. 8,2834 chains.

2. The length of a parallelogram is 12 chains, and the breadth 8 chains 74 links—What must be the breadth, the length being the same, to contain 4 acres? Ans. 3⅓ chains.

PROBLEM II.

The sides of a parallelogram are given, to cut off a given area from either of the angles. *Rule*—As half the area of the parallelogram is to the square of its given side, so is the area left at the verticle angle to the square of its similar side.

EXAMPLES.

1. The length of the sides of a parallelogram is 18

chains, and 12 chains 48 links—What is the length of the sides of a triangle, cut off from either of the angles of the parallelogram, containing 7 acres?

Ans. 14, 21, and 9,85 chains, the length of the required sides.

2. How many rods from either angle of a rectangular parallelogram must I measure to have 4½ acres, supposing the area of the parallelogram is 12 acres and 3 roods, and the shortest side is 5 chains?

Ans. 85,7 rods the longest, and 16,8 the shortest side.

3. How many chains from either angle of a parallelogram, whose sides are in the proportion of 2 to 3, must I measure to cut off 3 acres, provided the parallelogram contains 9 acres, 2 roods, and 16 rods?

Ans. 6,708 and 4,472 chains.

OF THE DIVISION OF TRIANGLES.

PROBLEM I.

The base and perpendicular of a right-angled triangle are given, to divide into two or more parts by lines parallel to either base or perpendicular, to find the length of the base or perpendicular. *Rule*—As the whole area is to the square of the given side, so is the area left at the angular point to the square of the required side, the square root of which will be the side.

EXAMPLES.

1. The base of a right-angled triangle is 20 chains, and the perpendicular 12 chains and 14 links, to be divided into two equal parts by a line parallel to the perpendicular—Required the length of the base of each part. Ans. 14,1421 length of the greater base, and 5,8579 — the less.

2. The base of a right-angled triangle is 18 rods, and the perpendicular 12 rods, to be divided into three equal parts by lines parallel to the base—Required the length of the perpendicular of each part.

Ans. 6,921 —, 2,877 —, and 2,202 + rods.

3. The base of a right angled triangle is 24 chains, and the perpendicular is 18 chains, to be divided into three parts by lines parallel to the perpendicular in the proportion of 2, 3, 4—Required the length of the base of each part. Ans. 11,3137, 6,5753 and 6,111 chains.

PROBLEM II.

The area of any triangle being given, to be divided into two or more parts. *Rule*—As the whole area is to the square of either side, so is the area left at either angle to the square of its similar side.

EXAMPLES.

1. The side of an equilateral triangle is 12 chains, to be divided into three equal parts by lines parallel to either side—Required the length of each part from the verticle angle.

Ans. 6,921, 9,798 chains, the sides required.

2. The three sides of a triangle are 12, 18, and 20 rods, to be divided into two equal parts by lines parallel to the base or longest side—Required the length of each part from the angle opposite to the base.

Ans. 8,485, and consequently the other part 3,515 rods the other.

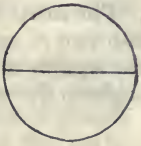
3. The three sides of a scalene triangle are in the proportion of 4, 5, and 7, containing 21 acres, to be divided into three parts by lines parallel to the longest side, the parts being in the proportion of 2, 3, and 5—Required the perpendicular distances asunder of those lines. Ans. 3,9806 and 3,9322 chains.

4. The base and perpendicular of a scalene triangle are in the proportion of 3 to 2, and the area is 12 acres—What must be the perpendicular distances asunder of the lines which will divide the triangle into three equal parts, those lines being parallel to the base?

Ans. 2,321, and 3,025, the distances required.

SECTION II.

OF CIRCLES.



A circle is a plane, bounded by a line which is equally distant from a point within called the centre. The bounding line is called the circumference or periphery. An arc is any portion of that circumference. A semicircle is half, and a quadrant is a quarter of a circle. The diameter is a straight line drawn through the centre, and terminated both ways by the circumference.

PROBLEM I.

To find the circumference of a circle, the diameter being given. *Rule*—Multiply the diameter by 3,1416, and the product will be the circumference. Or, as 113 is to the diameter, so is 355 to the circumference.

Note.—The above proportions are very near the truth, and are those in general use, the first being about as much too large as the second is too small. Its exactness cannot be found.

EXAMPLES.

1. What is the circumference of a circle whose diameter is 12 rods? Ans. 37,6992 rods.

2. What is the circumference of a circle whose diameter is 18 chains and 30 links? Ans. 57,49128 chains.

3. What is the circumference of a wheel whose diameter is 5 feet 2 inches? Ans. 16,2316 feet.

PROBLEM II.

The circumference given to find the diameter. *Rule*—Multiply the circumference by 113, and divide the product by 355; the quotient will be the diameter. Or, divide the given circumference by 3,1416; the quotient will be the diameter.

EXAMPLES.

1. The circumference of a circle is 16 chains—What is its diameter? Ans. 5,0929 + chains.

2. The wheel of a perambulator turns once and a half in a rod—What is its diameter? Ans. 3,5 + feet.

3. If the circumference of a carriage wheel be 16 feet 6 inches, what is its diameter? Ans. 5,2521 feet.

PROBLEM III.

The diameter given, to find the area. *Rule*—Multiply the square of the diameter by ,7854, and the product will be the area.

EXAMPLES.

1. The diameter of a circle is 16 chains—How many acres are contained within its periphery?
Ans. 20 acres, 0 roods, 17 perches.

2. The diameter of a circle is 18 rods—How many acres does it contain? Ans. 1,59 + acres.

3. How many square feet are contained in a circle whose diameter is 4 feet 3 inches?
Ans. 14,1862875 square feet.

4. What is the value of a circular garden whose diameter is 6 rods, at the rate of 8 cents per square foot?
Ans. 615,816432 dollars.

PROBLEM IV.

The area of a circle given, to find the diameter. *Rule*—Divide the area by the decimal ,7854, and extract the square root of the quotient.

EXAMPLES.

1. The area of a circle is 5 acres, 3 roods, and 26 rods—What is its diameter? Ans. 34,7 rods.

2. The area of a circle is 5 square chains—What is its diameter? Ans. 2.523 + chains.

3. What is the length of a rope, fastened to a stake in the centre of a circle, and the other end to the nose of a horse, which will permit him to feed on 2 acres of land? Ans. 2,5231 chains.

4. A circle contains 1¾ acres of land, worth 361 dollars per acre—How many dollars, allowing each to be 1½ inches in diameter, will reach across the circle through the centre. Ans. 2492,3184 dollars.

PROBLEM V.

The circumference given, to find the area. *Rule*—Multiply the square of the circumference by the decimal ,07958, and the product will be the area.

EXAMPLES.

1. The circumference of a circle is 24 chains—How many acres does it contain?
Ans. 4 acres, 2 roods, 13 perches.

2. The circumference of a circle is 120 rods—How many acres does it contain? Ans. 7,1622 acres.

3. The circumference of a circle is one rod—What is its area? Ans. 21,665 square feet.

PROBLEM VI.

The area of a circle is given, to find the circumference. *Rule*—Divide the area by the decimal ,07958, and the square root of the quotient will be the circumference.

EXAMPLES.

1. The area of a circle being 2 acres, 3 roods and 12 perches, what is the circumference?
Ans. 75,3657 + rods.

2. The area of a circular garden being one acre, what is the length of a stone wall which will enclose it?
Ans 44,839 + rods.

3. The value of a circular fish pond, at 4 dollars per square rod, is $46,50—How many dollars will encircle it, if the diameter of a dollar be 1½ inches?
Ans. 1595,3916 dollars.

PROBLEM VII.

To find the length of any arc of a circle. *Rule*—As 360 degrees are to the circumference of the circle, so is the number of degrees contained in the arc to the length of the arc required.

EXAMPLES.

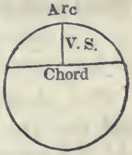
1. What is the length of an arc of 16 degrees, in a circle whose diameter is 100 feet? Ans. 13,9626 feet.

2. The diameter of a circle is 36 feet 3 inches—What is the length of an arc of 30 degrees? Ans. 9,49 + feet.

3. The circumference of a circle is 24 chains and 64 links—What is the length of an arc of 41 degrees?
Ans. 2,8062 + chains.

OF THE CHORD.

A chord is a straight line which joins the two extremities of an arc. The versed sine of an arc is that part of the diameter contained between the chord and the arc.



PROBLEM VIII.

The chord and versed sine given, to find the diameter of the circle. *Rule*—Divide the square of half the chord by the versed sine, and to the quotient add the versed sine; the sum will be the diameter of the circle.

EXAMPLES.

1. The chord of an arc of a circle is 12 feet, and the versed sine is 2—What is the diameter of the circle?
Ans. 20 feet.

2. The chord of the arc of a circle is 2 chains and 16 links, and the versed sine is 80 links—What is the diameter of the circle? Ans. 2,258 chains.

3. The chord of the arc of a circle is 5 rods, and the versed sine is 2 rods—What is the diameter of the circle? Ans. 5,125 rods.

PROBLEM IX.

The versed sine of an arc and the diameter of a circle given, to find the chord. *Rule*—From the diameter subtract the versed sine; multiply the remainder by the versed sine; the product will be the square of half the chord, the square root of which will give the half chord.

EXAMPLES.

1. The versed sine of an arc is 7 inches, and the diameter of the circle 4 feet 6 inches—What is the length of the chord?

Ans. 3,023 feet, the length of the chord.

2. The diameter of a circle is 18 feet, and the versed sine 2 feet—What is the length of the chord?

Ans. 11,3136.

3. The diameter of a circle is 30 chains, and the versed sine 3 chains—What is the length of the chord?

Ans. 18 chains.

PROBLEM X.

The chord and versed sine of an arc are given, to find the arc. *Rule*—To the square of half the chord add the square of the versed sine, and the square root of their sum will be the length of a diagonal line; to this diagonal add $\frac{5}{14}$ of the difference between it and half the chord, and the sum will be the length of half the arc, the double of which will be the length of the arc.

EXAMPLES.

1. The chord of an arc is 48 feet, and the versed sine 18 feet—Required the length of the arc.

Ans. 64,286 feet.

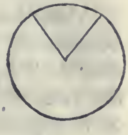
2. The chord of an arc is 40, and the versed sine 15—What is the length of the arc? Ans. 53,5714.

3. The diameter of a circle is 36, and the versed sine 2—What is the length of the arc? Ans. 16,8192.

4. The versed sine of an arc is 12, and the diameter of the circle 60—What is the length of the arc?

Ans. 55,689.

OF THE SECTOR.



A sector is a space contained between an arc and the two radii drawn from the extremities of the arc; it may be less or greater than a semicircle.

PROBLEM XI.

The chord and versed sine given, to find the area of a sector. *Rule*—Find the length of the half arc according to the preceding rule, and multiply it by the radii, or semi-diameter, and the product will be the area. Or if the degrees of the arc be given, say as 360 degrees are to the circumference of the circle, so are the number of degrees contained in the arc of the sector to the length of the arc; or as 360 degrees are to the area of the circle, so are the number of degrees contained in the arc of the sector to the area.

EXAMPLES.

1. The chord and versed sine of a sector are 40 and 15—What is the area of the sector? Ans. 558,0354.

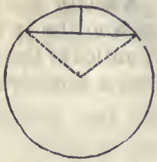
2. The diameter of a circle is 50 inches, and the versed sine 12—What is the area of the sector?
Ans. 640,4 square inches.

3. The versed sine of a sector is 18 inches, and half the chord of the arc 30 inches—What is the area of the sector? Ans. 1250,7138 inches.

4. The diameter of a circle is 36 feet, and the number of degrees contained in the arc of the sector 24—What is the area of the sector? Ans. 67,8585 +.

5. The diameter of a circle is 28 chains, and the number of degrees contained in the arc of the sector 124—What is the area of the sector? Ans. 212,0929 +.

OF THE SEGMENT.



A segment of a circle is the space contained between an arc and its chord. The chord is sometimes called the base of the segment. The height of a segment or versed sine, is the perpendicular from the middle of the base to the arc.

PROBLEM XII.

To find the area of a segment, the chord and versed sine being given.

Rule 1.—Find by the preceding rule or rules, the area of the sector, then from the semi-diameter of the circle subtract the versed sine or height of the segment, and multiply the half chord by the remainder, subtract this product from the area of the sector, and the remainder will be the area of the segment if less than a semicircle, but if greater, the last product must be added to the area of the sector.

EXAMPLES.

1. The chord of the segment of a circle 20 chains, and the versed sine or height of the segment 4 chains —What is the area of the segment?

Ans. 55,1525 square chains.

2. The diameter of a circle is 30 rods, and the versed sine of a segment cut off by a plane is 6 rods—What is the area of the segment? Ans. 127,836 square rods.

3. The diameter of a circle is 18 feet, and the chord of the segment is 12 feet—What is the area of the segment? Ans. 18,9057 square feet.

Rule 2.—Divide the height or versed sine by the diameter, and find the quotient in the table of versed sines; multiply the number on the right hand of the versed sine by the square of the diameter, and the product will be the area of the segment.

Note.—When the quotient arising from the versed sine, divided by the diameter, has a remainder or fraction after the third place of decimals, having taken the area answering to the first three figures, subtract it from the next following area; multiply the remainder by the said fraction and add the product to the first area, and the sum will be the area for the whole quotient.

EXAMPLES.

1. If the chord of a circular segment be 40, and its versed sine 10, what is its area? Ans. 279,5575.

2. If the diameter of a circle be 52, and the versed sine 2, what is the area of the segment? Ans. 26,9048 +.

3. The diameter of a circle is 40 chains, and the versed sine 10—What is its area? Ans. 245,6736.

4. The versed sine of the segment of a circle is 9 chains, and the diameter 100—How many acres does it contain? Ans. 35,011 acres.

5. The chord of the segment of a circle is 20 rods, and the versed sine 4—What is the area of the segment? Ans. 55,0014 rods.

6. The chord of the segment of a circle is 30 rods, and the versed sine 5—What is the area of the segment? Ans. 102,1875 sqr. rods.

PROBLEM XIII.

The diameter of a circle and the area of the segment given, to find the chord and versed sine of the segment.

Rule—Divide the given area by the square of the diameter, and the quotient will be the tabular area; and in the column to the left hand of the tabular area will be found the corresponding versed sine. Multiply the tabular versed sine by the given diameter, and the product will be the versed sine required; then from the diameter subtract the versed sine, and multiply the remainder by the versed sine; the product will be the square of half the chord, the square root of which will be the chord required.

EXAMPLES.

1. What is the length of the chord and versed sine of a segment containing 6 acres, cut off from a circle whose diameter is 20 chains?

Ans. 4,92 the versed sine, 17,227 chains the chord.

2. What is the length of a chord which will cut off $\frac{1}{3}$ of the area of a circle whose diameter is 50 chains?

Ans. 48,216 chains the length of the chord, and 18,386 the versed sine.

3. If the diameter of a circle be 80 rods, what is the length of the chord, and likewise the versed sine which cuts off $\frac{1}{4}$ of the area?

Ans. 23,84 rods the versed sine, and 73,18 the chord required.

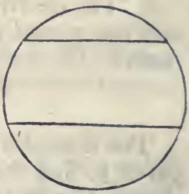
4. How deep must I saw into a round log, 36 inches diameter, to cut it $\frac{1}{3}$ off? Ans. 13,23792 inches.

5. One-fifth of the area is to be cut off from a circle whose diameter is 60 rods—What must be the length of the chord and versed sine?

Ans. 15,24 rods versed sine, and 52,236 the chord.

OF THE CIRCULAR ZONE.

A circular zone is the space between two parallel chords. It is called the middle zone when the two chords are of equal length.



PROBLEM XIV.

To find the area of a circular zone. *Rule*—From the area of the whole circle subtract the area of the two segments on the sides of the zone, and the remainder will be its area.

EXAMPLES.

1. What is the area of the zone, if the chord of one segment be 8, and the versed sine of the other 4,8; the diameter being 12?

Ans. 41,5501, the area of the zone.

2. What is the area of the middle zone of a circle whose diameter is 25, and the versed sine of each segment 5? Ans. 351,0962,5 the area required.

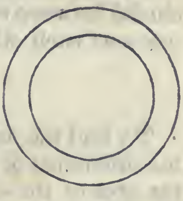
3. If the versed sine of one segment be 18 chains, and the other 15, and the diameter 50 chains, what is the area of the zone? Ans. 831,705 square chains.

4. If $\frac{1}{5}$ of the area be cut off in one segment, and the versed sine of the other be 10 feet, and the diameter of the circle be 25 feet, what is the remaining part worth at the rate of 20 cents per square foot? Ans. $41,868.

5. The area of a circle is 7,854 acres, and if $\frac{1}{4}$ of the diameter be cut off in one segment and $\frac{1}{5}$ in the other, what will the remainder or zone be worth at 25 cents per square rod? Ans. $39,7328.

OF THE CIRCULAR RING.

A circular ring is the space included between the two peripheries.



PROBLEM XV.

To find the area of a circular ring, or space included between the circumferences of two concentric circles. *Rule*—The difference between the areas of the two circles will be the area of the ring. Or, multiply the sum of the diameters by their difference, and this product by ,7854; it will give the required area.

EXAMPLES.

1. The diameter of the inner circle is 12 rods, and the outer 20—What is the area of the ring?
Ans. 201,0624 rods.

2. The diameter of the outer circle is 24 chains, and the inner 16 chains—What is the area of the ring?
Ans. 251,328 square chains.

3. The area of the outer circle contains 100 acres, and the diameter of the inner is equal to $\frac{2}{3}$ the diameter of the greater—What is the area of the ring?
Ans. 555,55 chains, area of the ring.

4. The circumference of the greater circle is 120 chains, and the diameter of the less is 28—What is the value of the ring at 8 cents per square rod?

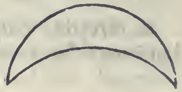
Ans. $678,828 +.

5. The diameter of the greater circle bears the same proportion to the less that 3 bears to 2, and the area of the ring is 2 acres—What is the diameter of each circle? Ans. 6,7704 chains the greater diameter, and 4,5136 the less.

6. The area of the inner circle bears such proportion to the area of the ring as 3 bears to 4, and the diameter of the less circle is 20 rods—What is the diameter of the greater? Ans. 30,55 + rods and

OF THE LUNE, OR CRESCENT.

A lune, or crescent, is the space included between two circular arcs which intersect each other.



PROBLEM XVI.

To find the area of a lune, or crescent. *Rule*—Find the difference of the two segments which are between the arcs of the crescent and its chord; that is, find the area of each segment, and the difference will be the area of the lune.

EXAMPLES.

1. The chord of the two segments is 72, and the height of the segments are, the less 20, and the greater 30—What is the area of the crescent?

Ans. 614,8692, the area of the lune.

2. The length of the chord is 40, the height of the less segment 4, and the height of the greater 10—What is the area of the lune?

Ans. 172,0465, area of the lune.

3. The length of the chord is 24, and the heights of the segments 10 and 2—Required the area of the lune.

Ans. 148,2374 + the area required.

4. The length of the chord is 48, and the heights of the segments 18 and 12—What is the area of the lune?

Ans. 233,8122, the area required.

5. The length of the chord is 50, and the heights of the segments 18 and 15—Required the area of the lune.

Ans. 134,9435.

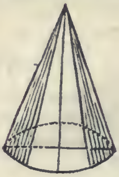
Note.—If semicircles be described on the three sides of a right-angled triangle as diameters, the two lunes formed on the base and perpendicular taken together will be equal to the area of the triangle.

SECTION III.

CONIC SECTIONS

DEFINITIONS.

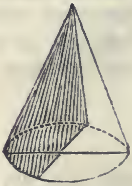
1. The Conic Sections are such plain figures as are formed by the cutting of a cone.



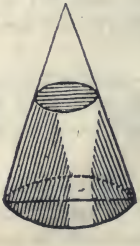
2. A cone is a solid, described by the revolution of a right-angled triangle about one of its legs which remains fixed.

3. The axis of the cone, is the right line about which the triangle revolves.

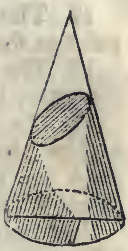
4. The base of a cone, is the circle which is described by the revolving leg of the triangle.



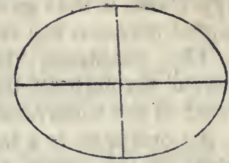
5. If a cone be cut through the vertex by a plane perpendicular to the base, the section will be a triangle.



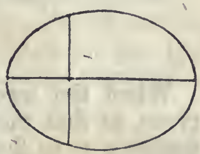
6. If a cone be cut into two parts by a plane parallel to the base, the section will be a circle.



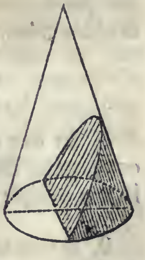
7. If a cone be cut by a plane which passes through its two slant sides in an oblique direction the section will be an ellipsis.



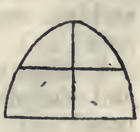
8. If two lines be drawn through the centre of an ellipsis at right angles with each other, and terminated both ways by the circumference, they are called the transverse, and conjugate diameters or axis; the longest line is the transverse axis, and the shortest the conjugate.



9. An ordinate, is a right line drawn from any point of the curve perpendicular to either of the diameters; and the abscissa is that part of the diameter which is contained between the vertex and the ordinate.



10. If a cone be cut by a plane which is parallel to either of its slant sides, the section will be a parabola.

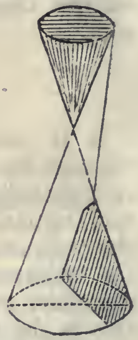


11. The axis of a parabola, is a right line drawn from the vertex so as to divide the figure into two equal parts.

12. The ordinate is a right line drawn from any point in the curve perpendicular to the axis.

13. The abscissa is that part of the axis which is contained between the vertex and the ordinate.

14. If a cone be cut into two parts by a plane, which being continued would meet the opposite cone, the section is called a hyperbola.



The transverse diameter, or axis of a hyperbola, is that part of the axis intercepted between the two opposite cones.

The conjugate diameter, is a line drawn throughout the centre perpendicular to the transverse.

An ordinate, is a line drawn from any point in the curve perpendicular to the axis; and the abscissa, is the distance intercepted between that ordinate and the vertex.

PROBLEM I.

When the transverse, conjugate and abscissa are given, to find the ordinate. *Rule*—As the transverse diameter is to the conjugate, so is the square root of the product of the two abscissas to the ordinate which divides them.

EXAMPLES.

1. In the ellipsis the transverse diameter is 120, the conjugate diameter is 40, and the abscissa 24—What is the length of the ordinate?

Ans. 16 the ordinate required.

2. If the transverse diameter be 40, the conjugate 30, and the abscissa 24, what is the length of the ordinate? Ans. 14,697 the ordinate required.

3. If the transverse diameter be 50, the conjugate 30, and the abscissa 18, what is the length of the ordinate? Ans. 14,4 the ordinate required.

PROBLEM II.

When the transverse, conjugate and ordinate are given, to find the abscissa. *Rule*—As the conjugate diameter is to the transverse, so is the square root of the difference of the squares of the ordinate and semi-conjugate to the distance between the ordinate and centre; this distance being added to and subtracted from the semi-transverse, will give the two abscissas required.

EXAMPLES.

1. The transverse diameter is 120, the conjugate diameter 40, and the ordinate 16—What is the length of each of the two abscissas?

Ans. 96 and 24 the abscissas required.

2. If the two diameters of an ellipse be 35 and 25, and the ordinate 10, what is the length of the two abscissas? Ans. 28 and 7, the abscissas required.

3. If the two diameters of an ellipse be 60 and 40, and the ordinate 12, what is the length of the two abscissas? Ans. 54 and 6, the abscissas required.

PROBLEM III.

When the conjugate, ordinate and abscissa are given, to find the transverse diameter. *Rule*—To or from the semi-conjugate, according as the greater or less abscissa is used, add or subtract the square root of the difference of the squares of the ordinate and semi-conjugate; then as this sum or difference is to the abscissa, so is the conjugate to the transverse diameter.

EXAMPLES.

1. The conjugate diameter is 40, the ordinate is 16, and the abscissa 24—Required the transverse diameter.

Ans. 120.

2. The conjugate diameter is 60, the ordinate 18, and the abscissa 24—Required the transverse diameter.

Ans. 240.

3. The ordinate is 24, and conjugate diameter 60, and the abscissa 36—What is the transverse diameter?
Ans. 180.

PROBLEM IV.

The transverse, ordinate and abscissa being given, to find the conjugate. *Rule*—As the square root of the product of the two abscissas is to the ordinate, so is the transverse diameter to the conjugate.

EXAMPLES.

1. The transverse diameter is 120, the ordinate 16, and the abscissa 24—What is the conjugate diameter?
Ans. 40.

2. The transverse diameter is 82, the ordinate 8, and the abscissa 32—What is the conjugate diameter?
Ans. 16,4, the conjugate required.

3. The transverse diameter is 35, the ordinate 10, and its abscissa 7—What is the conjugate?
Ans. 25, the conjugate.

PROBLEM V.

The transverse and conjugate diameters are given, to find the circumference. *Rule*—Multiply the square root of half the sum of the squares of the two diameters by 3,1416, and the product will be the circumference nearly.

EXAMPLES.

1. The transverse diameter of an ellipse is 24, and the conjugate 20—Required its circumference.
Ans. 69,398 —.

2. The axes of an ellipse are 60 and 40—What is the circumference? Ans. 160,2216, the circumference.

3. The transverse and conjugate diameters are 40 and 30—What is its circumference? Ans. 111,071.

PROBLEM VI.

The transverse and conjugate diameters are given, to find the area. *Rule*—Multiply the transverse and conjugate diameters together, and that product by ,7854; the last product will be the area.

EXAMPLES.

1. The transverse and conjugate diameters are 60 and 40 rods—How many acres does it contain?
Ans. 11,7185 acres.

2. The transverse and conjugate diameters are 40 and 36 chains—How many acres does it contain?
Ans. 113,0976 acres.

3. The transverse and conjugate diameters are 12 and 4 rods—How many square yards does it contain?
Ans. 1140,4 square yards.

PROBLEM VII.

The transverse and conjugate diameters given, to find the diameter of a circle containing the same area. *Rule*—Multiply the transverse and conjugate diameters together, and extract the square root of their product.

EXAMPLES.

1. The transverse and conjugate diameters are 36 and 24 chains—What is the diameter of a circle containing the same area? Ans. 29,3938 + chains.

2. The transverse and conjugate diameters are 49 and 16 rods—What is the diameter of a circle containing the same area? Ans. 28 rods.

PROBLEM VIII.

To find the area of an elliptical segment, whose base is parallel to either of the axes of the ellipse. *Rule*—Divide the height of the segment by the verticle axes, and the quotient will be the versed sine of the segment of a circle whose diameter is one; find the area of this segment, and multiply it by the product of the two diameters: this last product will be the area of the elliptical segment.

EXAMPLES.

1. What is the area of an elliptical segment whose height is 20, the vertical axis 80, and the parallel axis 50? Ans. 61,4184, the area required.

2. What is the area of an elliptical segment cut off parallel to the shorter axis, the height being 14 chains, and the two axes 25 and 35?

Ans. 256,697875 square chains.

3. What is the area of an elliptical segment whose height is 15 feet, the vertical axis 60, and the parallel axis 40 feet? Ans. 368,51 square feet.

PROBLEM IX.

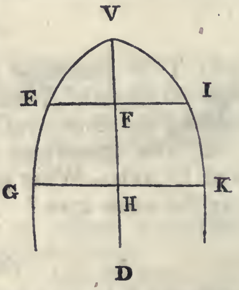
In a parabola, any three of the four following terms being given, viz. any two ordinates, and their two abscissas, to find the fourth. *Rule*—As any abscissa is to the square of its ordinate, so is any other abscissa to the square of its ordinate. Or, as the square root of any abscissa is to its ordinate, so is the square root of any other abscissa to its ordinate.

EXAMPLES.

1. The abscissa V F is 9, and its ordinate E F 6—Required the ordinate G H, whose abscissa V H is 16. Ans. 8, the ordinate.

2. The two abscissas are 9 and 16, and their corresponding ordinates 6 and 8—From any three of these to find the fourth.

Ans. 8 and 6.



PROBLEM X.

To find the length of any arc of a parabola, cut off by a double ordinate. *Rule*—To the square of the ordinate add $\frac{4}{3}$ of the square of the abscissa, and twice the square root of the sum will be the length of the curve required.

EXAMPLES.

1. The abscissa is 2, and its ordinate 6—What is the length of the arc G V K?

Ans. 12,8582, the length of the arc.

2. The abscissa is 3, and its ordinate 6—What is the length of the arc?

Ans. 13,8564, the length of the arc.

PROBLEM XI.

To find the area of a parabola, its base and height being given. *Rule*—Multiply the base by the height, and ⅔ of the product will be the area required.

EXAMPLES.

1. What is the area of a parabola whose height is 12, and the base or double ordinate is 16? Ans. 128.

2. The abscissa is 12, and the double ordinate 42—What is the area? Ans. 336.

3. The abscissa is 8, and the double ordinate 30—What is the area of the parabola? Ans. 160.

4. What is the area of a parabola whose abscissa is 10 and ordinate 18? Ans. 120.

PROBLEM XII.

To find the area of the frustrum of a parabola. *Rule*—Divide the difference of the cubes of the two ends of the frustrum by the difference of their squares; and this quotient, multiplied by ⅔ of the altitude, will give the required area.

EXAMPLES.

1. In the parabolic frustrum, the parallel ends are 6 and 10, and the altitude, or part of the abscissa, is 3—What is the area of the frustrum? Ans. 24,5, the area.

2. The greater end of the frustrum of a parabola is 24 and the less 20, and their distance 6—What is the area? Ans. 132,3636, the area required.

3. Required the area of a parabolic frustrum, the greater end of which is 10, the less 6, and the height 4,2. Ans. 34,3 the area required.

OF THE HYPERBOLA.

PROBLEM I.

In a hyperbola any three of the four following terms being given, viz. the transverse and conjugate diameters, an ordinate and its abscissa, to find the fourth.

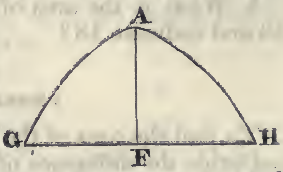
PROBLEM II.

The transverse and conjugate diameters, and the two abscissas being given, to find the ordinate. *Rule*—As the transverse diameter is to the conjugate, so is the square root of the product of the two abscissas to the required ordinate.

EXAMPLES.

1. In the hyperbola G A F the transverse diameter is 120, the conjugate 72, and the abscissa A H is 40—Required the ordinate F H.

Ans. 48, the ordinate.



2. The transverse diameter 24, the conjugate 21, and the less abscissa 8—What is the ordinate. Ans. 14.

3 The transverse of a hyperbola is 50, the conjugate 30, and the less abscissa 12—Required the ordinate. Ans. 16,3658 — the ordinate.

PROBLEM III.

The transverse and conjugate diameters and an ordinate being given, to find the two abscissas. *Rule*—As the conjugate diameter is to the transverse, so is the square root of the sum of the squares of the ordinate and semi-conjugate to the distance between the ordinate and the centre, or half the sum of the abscissas; then the sum of this distance and the semi-transverse will give the greater abscissa, and their difference the less.

EXAMPLES.

1. The transverse diameter is 120, the conjugate 72, and the ordinate 48—What are the two abscissas?
Ans. 160 the greater, and 40 the less.

2. The transverse and conjugate diameters are 24, and 21—Required the two abscissas to the ordinate 14.
Ans. 32 and 8, the abscissas required.

3. The transverse diameter is 60, and the conjugate 36—Required the two abscissas to the ordinate 24.
Ans. 80 and 20.

PROBLEM IV.

The transverse diameter, the two abscissas, and the ordinate being given, to find the conjugate. *Rule*—As the square root of the product of the two abscissas is to the ordinate, so is the transverse diameter to the conjugate.

EXAMPLES.

1. The transverse diameter is 60, the ordinate 24, and the two abscissas are 80 and 20—Required the conjugate diameter. Ans. 36.

2. The transverse diameter is 36, the ordinate 21, and the abscissas 12, and 48—Required the conjugate diameter. Ans. 31,5 the conjugate required.

PROBLEM V.

The conjugate diameter, the ordinate, and the two abscissas being given, to find the transverse. *Rule*—Add the square of the ordinate to the square of the semi-conjugate and find the square root of their sum. Take the sum or difference of the semi-conjugate and this root according as the less or greater abscissa is used, and then say—As the square of the ordinate is to the product of the abscissa and conjugate, so is the sum or difference above found to the transverse required.

EXAMPLES.

1. The conjugate diameter is 72, the ordinate is 48, and the less abscissa 40—What is the transverse diameter? Ans. 120 the transverse diameter.

2. The conjugate diameter is 21, the less abscissa 8, and its ordinate 14—Required the transverse diameter. Ans. 24 the diameter required.

3. Required the transverse diameter of the hyperbola whose conjugate is 36, the less abscissa 20, and the ordinate 24. Ans. 60 the transverse.

PROBLEM VI.

To find the length of an arc of a hyperbola beginning at the vertex. *Rule*—To 19 times the transverse add 21 times the parameter of the axis and multiply the sum by the abscissa, divided by the transverse—to each of the products thus found, add 15 times the parameter, and divide the former by the latter, then this quotient being multipled by the ordinate will give the length of the arc.

EXAMPLES.

1. The transverse diameter of a hyperbola is 80, the conjugate 60, the ordinate 10, and the abscissa 2,1637—Required the length of the arc. Ans. 10,3005 — the arc required.

2. The transverse diameter of a hyperbola is 120, the conjugate 72, the ordinate 48, and the abscissa 40—Required the length of the curve. Ans. 62,6448.

3. Required the whole length of the curve of a hyperbola to the ordinate 16, the transverse and conjugate axes being 80 and 60. Ans. 34,28672 the curve required.

PROBLEM VII.

To find the area of a hyperbola, the transverse and conjugate diameters and abscissa being given.

Rule 1.—To the product of the transverse and abscissa add $\frac{5}{7}$ of the square of the abscissa, and multiply the square root of the sum by 21.

Rule 2.—Add 4 times the square root of the product of the transverse and abscissa to the product last found, and divide the sum by 75.

Rule 3.—Divide 4 times the product of the conjugate and abscissa by the transverse, and this last quotient multiplied by the former will give the area.

EXAMPLES.

1. If the transverse and conjugate axes of a hyperbola be 30 and 18, and the abscissa or height be 10, what is the area? Ans. 151,68 the area.

2. If the transverse diameter be 100, the conjugate 60, and the less abscissa 50, what is the area of the hyperbola? Ans. 3220,3633584 the area required.

3. Required the area of the hyperbola to the abscissa 25, the two axes being 50 and 30.

Ans. 844,05432 the area required.

SECTION IV.

APPLICATION OF THE MENSURATION OF SUPERFICIES.

PROBLEM I.

THE side of a square being given, to find the diameter of a circle containing the same area. *Rule*—Multiply the side of the square by itself and divide the product by ,7854, the square root of the quotient will be the diameter.

EXAMPLES.

1. The side of a square is 16 chains—What is the diameter of a circle containing the same area?

Ans. 18,0539 + chains.

2. The side of a perfect square is 18 rods—How many yards is the diameter of a circle which contains the same area? Ans. 111,7094 + yds.

PROBLEM II.

The length and breadth of a rectangular parallelogram are given, to find the diameter of a circle that shall contain the same area. *Rule.*—Multiply the length of the parallelogram by its breadth, and divide the product by ,7854 and extract the square root of the quotient.

EXAMPLES.

1. The length and breadth of a parallelogram are 24 and 16 chains—What is the diameter of a circle which contains the same area?

Ans. 22,1116 — chains.

2. The length and breadth of a parallelogram are 32 and 18 rods—What is the diameter of a circle that contains the same area? Ans. 27,0816 +.

PROBLEM III.

The base and perpendicular of a right-angled triangle being given, to find the diameter of a circle which shall contain the same area. *Rule.*—Multiply half the base by the perpendicular and divide the product by ,7854, and extract the square root of the quotient.

EXAMPLES.

1. The base of a right-angled triangle is 17 rods, and the perpendicular 14—How many chains will be the diameter of a circle which contains the same area?
Ans. 3,077 chains.

2. The base and perpendicular of a right-angled triangle are 16 and 20 chains—How many rods will be the diameter of a circle which contains the same area? Ans. 57,092 —

PROBLEM IV.

The difference between the hypotenuse and perpendicular is given, and the length of the base, to find the diameter of a circle which will contain the same area. *Rule*—From the square of the base subtract the square of the difference, divide the remainder by twice the difference, and the quotient will be the perpendicular; multiply the base by half the perpendicular, divide the product by ,7854, and extract the square root of the quotient.

EXAMPLES.

1. The base of a right-angled triangle is 18 chains, and the hypotenuse is 6 rods longer than the perpendicular—How many rods will be the diameter of a circle which shall contain the same quantity?
Ans. 140,2279 rods.

2. The diameter of a circle is required which shall contain 4 times as much surface as a triangle whose base is 24 chains, and hypotenuse 25 rods longer than the perpendicular. Ans. 204,9472 +.

PROBLEM V.

The three sides of a scalene triangle given, to find the diameter of a circle equal in area thereto. *Rule*—Add the three sides of the scalene triangle together, and from their half sum subtract each side separately; multiply the half sum and these remainders continually together, and the square root will be the area; divide the area by ,7854, and the square root of the quotient will be the diameter.

EXAMPLES.

1. The three sides of a scalene triangle are 14, 18, and 24 chains—What is the diameter of a circle containing the same area? Ans. 12,6267 chains the diameter.

2. The three sides of a scalene triangle are 18, 20, and 26 rods—What is the diameter of a circle containing 3 times as much? Ans. 26,1916 + rods the diameter.

PROBLEM VI.

The three sides of a scalene triangle being given, to find a point equally distant from each of the angles, or the diameter of a circle circumscribing the triangle. *Rule*—As the whole length of the base is to the sum of the other two sides, so is the difference of those sides to the difference of the segments of the base; half the said difference added to half the base gives the greater of the two segments, subtracted leaves the less; then from the square of the shortest side subtract the square of the shortest segment, and the square root of the remainder will be the length of a perpendicular let fall from the angle opposite the base to the base: then, as this perpendicular is to the longest side except the base, so is the shortest side to the diameter of a circle whose periphery will pass through all the angular points; half that diameter will be the distance from each of the angles.

EXAMPLES.

1. The three sides of a scalene triangle are 6, 7, and 9 chains—How far from each of the angles must a building be placed to be equally distant from each of the angles? Ans. 4,50644 chains.

2. The three sides of a scalene triangle are 32, 40, and 60 rods—What is the diameter of that circle whose periphery will pass through all the angular points? Ans. 64,9 rods.

PROBLEM VII.

Having the transverse and conjugate diameters of an ellipsis given, to find the diameter of a circle containing the same area. *Rule*—Multiply the two diameters together, and extract the square root of the product.

EXAMPLES.

1. The transverse diameter of an ellipsis is 34 chains, and the conjugate 18—What is the diameter of a circle containing the same area? Ans. 24,7386 +.

2. The transverse and conjugate diameters are 18 and 12 rods—What is the diameter of a circle containing 5 times as much? Ans. 32,86335 + rods.

3. The two diameters of an ellipsis are 40 and 30 feet—How many yards will the diameter of a circle be that contains 3 times as much? Ans. 20 yards.

PROBLEM VIII.

To find the diameter of a circle circumscribing a square. *Rule*—Extract the square root of double the square of one side.

EXAMPLES.

1. The side of a square is 15 feet—What is the diameter of its circumscribing circle? Ans. 21,2132 +.

2. The side of a square is 21 chains—How many rods is the diameter of its circumscribing circle? Ans. 119,2 —.

PROBLEM IX.

The diameter of a circle being given, to find the side of its greatest inscribed square. *Rule*—Extract the square root of half the square of the diameter.

EXAMPLES.

1. The diameter of a circle is 18 chains—What is the side of its greatest inscribed square?
Ans. 12,7279 + chains.

2. The area of a circle is 25 acres—What is the area of its greatest inscribed square, and likewise the area of the 4 segments that reduce the circle to a square?
Ans. 159,15457 square chains the area—90,84543 the area of the segments; the length of the sides is 12,6156 + chains.

3. The circumference of a circle is 28,2744 chains—How many acres are contained in its inscribed square?
Ans. 4,05 acres.

PROBLEM X.

The length and breadth of a rectangular parallelogram are given, to find the diameter of its circumscribing circle. *Rule*—Extract the square root of the sum of the squares of the length and breadth.

EXAMPLES.

1. The length of a parallelogram is 16 feet, and the breadth 12—What is the diameter of its circumscribing circle? Ans. 20 feet.

2. The length of a rectangular parallelogram is 64 rods, and the breadth 48—What is the area and diameter of a circle circumscribing the parallelogram?
Ans. The diameter is 80 rods, and contains 31 acres, 1 rood, and 26,56 rods.

3. The length of a rectangular parallelogram is 24 chains, and the breadth 18—What is the area of its circumscribing circle, and likewise of the 4 segments contained between the sides of the parallelogram and the periphery of the circle?
Ans. 706,86 square chains the area of the circle, and 274,86 the area of the segments.

PROBLEM XI.

The length of the side of an equilateral triangle is given, to find the diameter of its circumscribing circle. *Rule*—Multiply the length of one side by the square root of the number 3, and the product will be three times the semidiameter; this product multiplied by 2, and divided by 3, the quotient will be the diameter.

EXAMPLES.

1. The length of the side of an equilateral triangle is 30 feet—What is the diameter of its circumscribing circle? Ans. 34,641 feet, the diameter.

2. The length of the side of an equilateral triangle is 36 chains—What is the diameter of its circumscribing circle? Ans. 41,5692 chains.

3. The length of the side of an equilateral triangle is 24 rods—What is the diameter of its circumscribing circle? Ans. 27,7128 rods.

PROBLEM XII.

The diameter of a circle is given, to find the side of the greatest equilateral triangle inscribed within its periphery. *Rule*—Divide three times the semidiameter by the square root of the number 3; the quotient will be the length of the side.

EXAMPLES.

1. The diameter of a circle is 24 feet—What is the length of the side of the greatest equilateral triangle inscribed within its periphery? Ans. 21,01556 — feet.

2. The diameter of a circle is 36 rods—What is the side of the greatest equilateral triangle that can be inscribed within its periphery?

Ans. 31,17635 + rods, the length of the side.

3. The circumference of a circle is 24 chains—What is the length of the side of the greatest equilateral triangle inscribed within it, and also the area of the segments which reduce it to a triangle?

Ans. 6,61593 + chains, the side required.

PROBLEM XIII.

Having the length of the three perpendiculars let fall from a point within an equilateral triangle to the opposite sides given, to find the diameter of a circle circumscribing the triangle. *Rule*—Add the three perpendiculars together, and divide their sum by the square root of the number 3; the quotient will be the length of half the side. Multiply this quotient by 2; the product will be the length of the side. You will then have the length of the side, to find the diameter. (See Prob. XI.)

EXAMPLES.

1. The length of 3 perpendiculars let fall from a point within an equilateral triangle are 10, 16, and 24 feet—What is the diameter of a circle circumscribing the triangle? Ans. 66,649 + feet the diameter required.

2. The length of three perpendiculars let fall from a point within an equilateral triangle are 16, 18, and 24 rods—What is the diameter of a circle circumscribing the triangle? Ans. 77,3329 rods the diameter.

3. The sum of three perpendiculars let fall from a point within an equilateral triangle is 102 rods—What is the area of a circle circumscribing the triangle, and also the area of the segments reducing it to a triangle?

Ans. The area of the segments 8519,9484 square rods, 14526,7854 square rods the area of the circle.

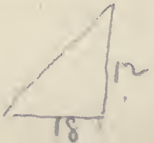
PROBLEM XIV.

The base and perpendicular of a right-angled triangle being given, to find the side of the greatest square that can be formed within the perimeter. *Rule*—Divide the product of the base and perpendicular by their sum, and the quotient will be the side of the greatest square inscribed.

EXAMPLES.

1 The base of a right-angled triangle is 18 feet, and the perpendicular 12 feet—What is the length of the side of its greatest inscribed square? Ans. 7,2 feet.

2. The perpendicular of a right-angled triangle is



18 rods, and the base 24—What is the length of the side of the greatest inscribed square?

Ans. 10,2857 + rods.

PROBLEM XV.

To determine the diameter of the greatest circle inscribed in a square, the area in the corners only given. *Rule*—Subtract ,7854 from one, then as that remainder is to the square of one, so is the area in the corners to the square of the diameter, the square root of which will be the diameter.

EXAMPLES.

1. The area in the corners of a square containing the greatest inscribed circle is 2 acres, 3 roods, and 6 rods—Required the diameter of the circle.

Ans. 45,5882 + rods the diameter required.

2. The area in the corners of a square containing its greatest inscribed circle is 36,2674 square rods—What is the diameter of the circle? Ans. 13 rods.

3. A certain square contains the greatest circle that can be described within its perimeter, and one acre in each corner—What is the area of the square and diameter of the circle? Ans. 13,6525 chains.

PROBLEM XVI.

Having the diameter of a circle given, to find another containing a proportionate quantity. *Rule*—Multiply the square of the given diameter by the given proportion, and the square root of the product will be the required diameter.

EXAMPLES.

1. The diameter of a circle is 24 chains—What is the diameter of one containing only ¼ of the area?

Ans. 12 chains.

2. The area of a circle is 21 acres and 3 roods, and contains another whose contents are 7 acres and 1 rood —What is the diameter of each, and the width of the ring between the peripheries?

Ans. 16,638 + chains the diameter of the greater circle, and 9,607 + the less, 3,5155 the width of the ring.

3. A carpenter is to put an oaken curb to a round well at 18 cents per square foot; the breadth of the curb is to be $7\frac{1}{2}$ inches, and the diameter within $3\frac{1}{2}$ feet—What will be the expense? Ans. $1,4454.

PROBLEM XVII.

The distance of the centres of two circles whose peripheries intersect each other being given, to find the area enclosed by the two. *Rule*—From the given diameter subtract the distance of the two centres, and half the remainder will be the versed sine or height of each of the segments; find the area of those segments and subtract it from the area of the two circles, and the remainder will be the area enclosed.

EXAMPLES.

1. The distance of the centres of two circles, whose diameters are each 50 rods, is 30 rods—What is the area of the space enclosed by their peripheries?

Ans. 3367,885 square rods.

2. The difference of the centres of two circles is 24 chains, and their diameters are each 40 chains—What is the area enclosed by their peripheries?

Ans. 2155,4464 square chains.

3. The diameters of two circles are each 20 feet, and their centres are 10 feet asunder—What is the area enclosed by their peripheries, and the length of the chord of each segment formed by their intersection?

Ans. 505,4832 square feet the area enclosed, and 17,32052 feet, length of the chord.

PROBLEM XVIII.

The diameters of two wheels of unequal sizes, and the length of the axletree on which they are placed, being given, to find the diameters of the circles made by their rolling on a plane. *Rule*—Subtract the less diameter from the greater, then as that remainder is to the length of the axletree, so is the diameter of the less wheel to a fourth term. To the square of this fourth term add the square of half the diameter of the less wheel, and the square root of the sum will be the semi-diameter of the less circle: if the diameter of the greater circle be required, the diameter of the greater wheel must be used instead of the less.

EXAMPLES.

1. If two wheels, the one four feet in diameter and the other five, be placed on an axletree 20 feet long, and set rolling on a plane till they describe a circle, what is the diameter of the circle made by the less wheel?

Ans. 160,05 feet the diameter.

2. If two wheels, the one three feet in diameter and the other five, be placed on an axletree 16 feet long, and set rolling till they describe a circle, what is the diameter of the circle made by the greater wheel?

Ans. 80,156 feet the diameter required.

3. The diameters of two wheels are three and four feet, and placed on an axletree 30 feet long—If set rolling on a plane till they describe a circle, what will be the area inclosed by each circle, and also the area of the ring between the circles?

Ans. 5027,955 square yards the area of the greater circle, 2828,487 the less, and 2199,468 area of the ring.

PROBLEM XIX.

When a vessel on the ocean is discovered from the mast head of another, level with the observer's eye, to find the distance between. *Rule*—To the earth's diameter in feet add the elevation of the eye, and multiply the sum by that elevation; the square root of the product will be the distance.

Note.—The diameter here used is 8000 miles, on account of its convenience.

EXAMPLES.

1. How far may a mountain be seen at sea which is a mile high, supposing the eye of the observer elevated 20 feet above the surface? Ans. 94,95 + miles.

2. A sailor at the masthead of a vessel at sea being 100 feet above the surface, discovers another level with the eye—What is the distance between them?

Ans. 12,309 + miles.

3. How far may the lamp of a lighthouse 150 feet high be seen at sea, when the eye of the observer is elevated 100 feet above the surface?

Ans. 27,091 — miles.

PROBLEM XX.

The perimiter of a square, and the area being equal, or in any given proportion, to find the side of the square. Or the circumference of a circle and area being equal, or in given proportion, to find the diameter. *Rule*—Suppose any convenient number for the side of the square or diameter of the circle, whichever may be required, and find its perimeter, and likewise the area; divide the perimeter by the area, and the quotient will be the side of the square or diameter of the circle, whichever may be required.

EXAMPLES.

1. The paving of a square at 2 shillings a yard, cost just as much as the enclosing it at 8—What was the length of the side of the square? Ans. 16 yards.

2. A gentleman purchased a garden in the form of a circle for 33 dollars a square rod, and to have as many square rods as he could encircle with dollars, each being one inch and a half in diameter, the dollars encompassing the same, exactly paying for the garden at the above rate—What must be its diameter?

Ans. 16 rods the diameter required.

3. A gentleman, two daughters had,
And both were very fair,
To each he gave a piece of land,
One round—the other square.
At forty dollars the acre just,
Each piece its value had,
The cents that did encompass each,
For each exactly paid.
If 'cross a cent be just an inch,
As it is very near,
Who had the better portion,
That had the round—or square?

Ans. 250,9056 dollars the value of the square, and 123,1632864 dollars value of the circle.

SECTION V.

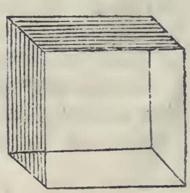
MENSURATION OF SOLIDS.

The measure of any solid body is the whole capacity or content of that body when considered under the triple dimensions of length, breadth, and thickness.

A cube whose side is one inch, one foot, or one yard, &c., is called the measuring unit, and the content or solidity of any figure is computed by the number of those cubes contained in that figure.

DEFINITION.

A cube is a solid bounded by six equal square sides.



PROBLEM I.

To find the area of the surface of a cube. *Rule*—Multiply the square of the length of one side by the number of sides and the product will be the area of the surface.

EXAMPLES.

1. The side of a cube is 18 inches—What is the area of its surface? Ans. 13,5 square feet.

2. The side of a cube is 25 inches—What is the area of its surface? Ans. $26\frac{1}{24}$ square feet.

PROBLEM II.

The area of the surface of a cube being given, to find the length of the side. *Rule*—Divide the area by 6, and extract the square root of the quotient.

EXAMPLES.

1. The area of a cube is 2400 square inches—What is the length of the side? Ans. 20 inches.

2. The area of a cube is 24 square feet—What is the length of its side? Ans. 2 feet.

PROBLEM III.

To find the solidity of a cube, the height of one of its sides being given. *Rule*—Cube the given side.

EXAMPLES.

1. What is the solidity of a cube, whose side is 3 feet? Ans. 27 solid feet.

2. How many solid feet in a cubic box whose depth is 32 inches? Ans. $18\frac{26}{27}$ solid feet.

3. How many cubic inches are contained in a cubical piece of timber, whose length is 42 inches?
Ans. 74088 cubic inches.

PROBLEM IV.

To find the side of a cube, the solidity being given. *Rule.*—Extract the cube root of the solidity.

EXAMPLES.

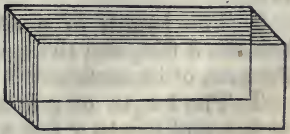
1. What is the length of the side of a cubic box containing 18 solid feet? Ans. 2,620741 feet.

2. What is the height of a cubic box containing 50 bushels?* Ans. 47,5 inches.

3. What is the height of a cubic box that contains 10 bushels? Ans. 27,454 + inches.

DEFINITION.

A parallelopipedon, is a solid bounded by six quadrilateral planes, every opposite two of which are equal and parallel.



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* A cubic foot is about eight tenths of a bushel, and is sufficiently exact for common uses.

PROBLEM V.

To find the solidity of a parallelopipedon. *Rule*—Multiply the length by the breadth, and that product by the depth or altitude, and it will give the solidity required.

EXAMPLES.

1. The length of a parallelopipedon is 8 feet, its breadth 3 feet, and thickness two feet—How many solid feet does it contain? Ans. 48 solid feet.

2. The length of a parallelopipedon is 36 inches, the width 20, and the depth 18 inches—How many solid feet will it contain? Ans. 7,5 solid feet.

3. How many bushels are contained in a bin 5 feet 6 inches long, 4 feet 9 inches wide, and 3 feet 9 inches deep? Ans. 78,724 — bushels.

4. What is the solidity of a block of marble whose length is 10 feet, breadth 5 feet 3 inches, and thickness 2 feet 6 inches? Ans. 131,25 solid feet.

PROBLEM VI.

To find the area of the surface of a parallelopipedon. *Rule*—Find the area of each side, and likewise of the ends, and their sum will be the area of the surface of the parallelopipedon.

Note.—The surface of a parallelopipedon is equal to the sum of the areas of each of its sides and ends.

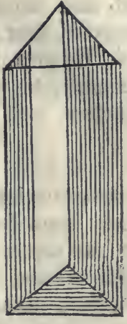
EXAMPLES.

1. The length of a parallelopipedon is 10 feet, the breadth 5, and the height 3—What is the area of its surface? Ans. 190 square feet.

2. The length of a stick of timber is 4 feet, the breadth 3 feet, and the height 2 feet 9 inches—What is the area of its surface? Ans. 62,5 square feet.

3. The length of a box is 8 feet, its width 4 feet 3 inches, and height 2 feet 6 inches—What is the area of its surface? Ans. 122,25 square feet.

DEFINITION.



A prism is a solid bounded by plane figures or faces, two of which are parallel, similar and equal, and the others are parallelograms.

PROBLEM VII.

To find the area of the surface of a prism. *Rule*—Add the area of the ends to the area of the sides, and their sum will be the area of the surface.

EXAMPLES.

1. What is the area of the surface of an equilateral triangular prism whose length or height is 12 feet, and either of its equilateral ends 2 feet 6 inches?
Ans. 95,4126 +.

2. The height of an equilateral triangular prism is 8 feet, and each side of its base is 14 inches—What is the area of its surface? Ans. 29,1787 square feet.

3. The height of an equilateral triangular prism is 9 feet 6 inches, and each side of its base is 24 inches—What is the area of its surface?
Ans. 60,4641 square feet.

PROBLEM VIII.

To find the solidity of a prism. *Rule*—Find the area of the base, and multiply it by the height, the product will be its solidity.

EXAMPLES.

1. Each side of the base of an equilateral triangular prism is 18 inches, and the height 40 feet—How many solid feet does it contain? Ans. 38,97 + solid feet.

2. How many solid feet are contained in an equilateral triangular prism whose height is 24 feet, and each side of whose base is 18 inches? Ans. 23,386 — feet.

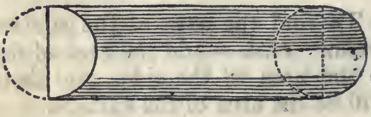
3. What is the value of a prism, whose height is 32

feet, and each side of the base 14 inches, at 20 cents per solid foot? Ans. $3,772 +.

4. Required the solidity of a prism whose base is a hexagon, supposing each of the equal sides to be 1 foot 6 inches, and the length of the prism 16 feet?

Ans. 93,53 +.

DEFINITION.



A cylinder is a solid described by the revolution of a right-angled parallelogram about one of its sides which remains fixed.

PROBLEM IX.

To find the surface of a cylinder. *Rule*—Multiply the circumference of the end by the length, and the product will be the convex surface, to which add the area of each end, and the sum will be the whole surface of the cylinder.

EXAMPLES.

1. What is the convex surface of a right cylinder whose length is 20 feet, and the diameter of its end or base 2 feet? Ans. 125,664 square feet.

2. What is the whole surface of a right cylinder, the diameter of whose base is 30 inches, and the height 5 feet? Ans. 49,0875 square feet.

3. How many square feet in the whole surface of a right cylinder, whose length is 20 feet, and diameter 16 inches?

Ans. 86,5685 square feet, the area required.

4. How many square yards in the convex surface of a cylinder 10 feet long, and diameter 4 feet?

Ans. 13,9627 + square yards.

PROBLEM X.

To find the solidity of a cylinder. *Rule.*—Multiply the square of the diameter by ,7854, and that product by the length will give the solidity.

EXAMPLES.

1. The diameter of a cylinder is 16 inches, and the length 20 feet—What is its solidity?
Ans. 27,9253 + solid feet.

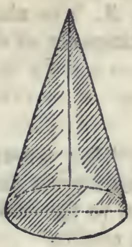
2. The length of a cylinder is 30 feet, and the diameter 20 inches—What is its solidity?
Ans. 65,45 solid feet.

3. The diameter of a cylinder is 10 inches, and the length 4 feet—What is its solidity?
Ans. 2,1816 + solid feet.

4. How many solid feet in a round stick of timber 16 feet long, and the diameter at each end 15 inches?
Ans. 19,635 solid feet.

DEFINITION.

A cone is a round solid body of a true taper from the base to a point, which is called the vertex.



PROBLEM XI.

To find the whole surface of a right cone. *Rule*—Multiply the circumference of the base by the slant, height, or the length of the side of the cone, and half the product will be the area of the convex surface, to which add the area of the base, and the sum will be the whole surface of the cone.

EXAMPLES.

1. The diameter of the base of a right cone is 3 feet, and the slant height 15 feet—Required the convex surface of the cone. Ans. 72,686 square feet.

2. The slant height of a right cone is 20 feet, and the diameter three feet—What is the whole surface of the cone? Ans. 101,3166 square feet.

3. The circumference of a right cone is 10 feet, and the perpendicular height 12—What is the convex surface of the cone? Ans. 60,525 square feet.

PROBLEM XII.

To find the solidity of a cone, the diameter of the base and perpendicular altitude given. *Rule*—Multiply the square of the diameter of the base by ,7854, and that product by ⅓ of the perpendicular altitude, the product will be the solidity.

EXAMPLES.

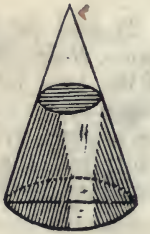
1. The diameter of the base of a cone is 3 feet 6 inches, and the perpendicular altitude 9 feet—What is its solidity? Ans. 28,86345 solid feet.

2. The circumference of the base of a cone is 10 feet, and the perpendicular altitude 12 feet—What is its solidity? Ans. 31,829 + cubic feet.

3. The slant side of a right cone is 10 feet, and the diameter of the base 6 feet—What is its solidity?
Ans. 89,9069 + cubic feet the solidity.

DEFINITION.

The frustrum of a cone, is a right cone with the top cut off by a line parallel to the base, and is the usual form of a cistern.



PROBLEM XIII.

To find the convex surface of the frustrum of a cone. *Rule*—Multiply the sum of the peripheries of the two ends by the slant height of the frustrum, and half the product will be the area of the convex surface required.

EXAMPLES.

1. The circumference of the base of the frustrum of a cone is 15,75 feet, and that of the top 22,25, and the slant height is 6 feet—What is the area of the convex surface? Ans. 114 square feet.

2. The diameter of the base of the frustrum of a cone

is 4 feet, and that of the top 3, the slant side being 5 feet—What is the area of the convex surface?

Ans. 54,978 square feet.

3. The greatest diameter of the frustrum of a cone is 5 feet, and the less 4, the perpendicular height 6 feet—Required the convex surface.

Ans. 84,8232 square feet.

PROBLEM XIV.

To find the solidity of the frustrum of a cone. *Rule*—Multiply the top and bottom diameters together, and to their product add $\frac{1}{3}$ of the square of the difference of the two diameters; multiply that sum by ,7854, and that product by the height—the last product will be the solidity.

EXAMPLES.

1. The two diameters of the frustrum of a cone are 5, and 3 feet, and the height 4 feet—What is the solidity? Ans. 51,3126 + cubic feet.

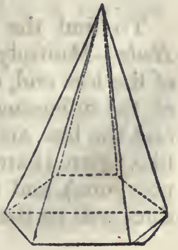
2. The top of a cistern is 5 feet 6 inches, and the bottom 4 feet 9 inches, the height or depth 7 feet 3 inches—How many hogsheads will it contain?

Ans. 17,79 +.

3. If the top diameter of a cistern be 5 feet, the bottom 4 feet 9 inches, and the depth 8 feet, how many hogsheads will it contain? Ans. 17,734 + hogsheads.

DEFINITION.

A Pyramid is a solid whose sides are all triangles, meeting in a point at the vertex, and the base any plane figure whatever. Pyramids receive their names according to the various forms of their bases: if it has 3 sides it is called a triangular pyramid; if 4, it is called a square or quadrangular pyramid; if 5 equal sides, a pentagonal pyramid, &c.



PROBLEM XV.

To find the solidity of a pyramid. *Rule*—Find the area of the base according to its figure by superficial measure, and multiply that product by $\frac{1}{3}$ of its perpendicular altitude, the product will be its solidity.

EXAMPLES.

1. What is the solidity of an equilateral triangular pyramid each side of whose base is 20 inches, and perpendicular height 18 feet? Ans. 7,2169 — cubic feet.

2. What is the solidity of a square pyramid whose perpendicular altitude is 6 feet, and each side of whose base is 2 feet? Ans. 8 cubic feet.

3. What is the solidity of a pentagonal pyramid, each side of the base being 30 inches, and perpendicular altitude 12 feet? Ans. 43,012 — cubic feet.

4. What is the solidity of a hexagonal pyramid, each side of the base being 20 inches, and perpendicular altitude 9 feet 9 inches?
Ans. 23,4548 + cubic feet.

5. Each side of the base of an octagonal pyramid is 20 inches, and the perpendicular altitude 12 feet—What is its solidity? Ans. 53,6492 — cubic feet.

DEFINITION.

The frustrum is any part of a pyramid cut off by a plane parallel to its base.

PROBLEM XVI.

To find the solidity of the frustrum of a pyramid. *Rule*—Multiply the side of the greater end by the side of the less end, and to their product add $\frac{1}{3}$ of the square of the difference between the side of the greater end and the less, and multiply the sum by the multiplier of like figure (found in the table of multipliers for regular polygons), and that product by the height; the last product will be the solidity.

EXAMPLES.

1. What is the solidity of the frustrum of an equilateral triangular pyramid, the base being 14 inches on a side, and the top 8, the height 10 feet.

Ans. 3,7287 + cubic feet.

2. The height of the frustrum of a square pyramid is 8 feet each side, the base 16 inches, and the top 10 inches—Required the solidity of the frustrum.

Ans. 9,55 + cubic feet.

3. The altitude of the frustrum of a pentagonal pyramid is 4 feet 6 inches, the side of the greater end 18 inches, and of the less 12—What is the solidity of the frustrum?

Ans. 12,2581 + cubic feet.

4. Each side of the base of a hexagonal frustrum is 10 inches, and of the top 7 inches, the height being 5 feet—What is the solidity of the frustrum?

Ans. 6,5854 + cubic feet.

DEFINITION.

A wedge is a solid of five sides, viz. a rectangular base, two rhomboidal sides meeting in an edge, and two triangular ends. The height of the wedge is a perpendicular drawn from any point in the edge to the plane of the base.

PROBLEM XVII.

To find the solidity of a wedge. *Rule*—Add the length of the edge to twice the length of the base, and multiply the sum by the height of the wedge, and that product by the breadth of the base, and $\frac{1}{6}$ of the last product will be the solidity.

EXAMPLES.

1. How many solid feet are there in a wedge whose base is 5 feet 3 inches long, and 9 inches broad, the length of the edge being 3 feet 6 inches, and the perpendicular height 2 feet 3 inches?

Ans. 3,9375 cubic feet.

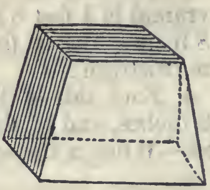
2. The length and breadth of the base of a wedge are 35 and 15 inches, and the length of the edge is 55 inches—What is the solidity, supposing the perpendicular height to be 18 inches?

Ans. 3,2552 + cubic feet.

3. If the base of a wedge be 27 inches by 8, the edge 36, and the perpendicular 3 feet 6 inches, what is the solidity? Ans. 2,9166 + cubic feet.

DEFINITION.

A Prismoid is a solid whose ends or bases are parallel, but not similar, and whose sides are quadrilateral. It differs from a prism or a frustrum of a pyramid, in having its ends dissimilar. It is a rectangular prismoid when its ends are right parallelograms.



PROBLEM XVIII.

To find the solidity of a prismoid. *Rule*—To the sum of the areas of the two ends, add four times the area of a section parallel to, and equally distant from, both ends, and this last sum multiplied by $\frac{1}{6}$ of the height will give the solidity.

EXAMPLES.

1. What is the solidity of a rectangular prismoid, the length and breadth of one end being 14 and 12 inches, and the corresponding sides of the other 6 and 4 inches, and the perpendicular 30 feet 6 inches?

Ans. 18,074 + cubic feet.

2. What is the solidity of a stick of hewn timber, whose ends are 30 inches by 27, and 24 by 18, and whose length is 48 feet? Ans. 204 cubic feet.

3. What is the solid content of a stick of hewn timber, whose greater end measures 12 inches by 8, the less end 8 inches by 6, and the length 5 feet?

Ans. 2,4537 cubic feet.

4. What is the capacity of a coal waggon whose inside dimensions are as follow: at the top the length is 7 feet, and breadth 6 feet; at the bottom the length is 5 feet and the breadth 3 feet, and the perpendicular depth 4 feet? Ans. 54 cubic feet.

SECTION VI.

OF SURFACES AND SOLIDS.

DEFINITION.

A Sphere or Globe is a round solid body, in the centre of which is a point from which all right lines drawn to the surface are equal.

PROBLEM I.

To find the convex surface of a sphere or globe *Rule*—Multiply the square of the diameter by 3,1416, the product will be the area of the convex surface.

EXAMPLES.

1. What is the area of the convex surface of a sphere whose diameter is 10 feet?

Ans. 314,16 square feet.

2. What is the area of the convex surface of a globe whose diameter is 4 feet? Ans. 50,2656 square feet.

3. If the diameter of the globe we inhabit be 8000 miles, what is the area of its surface?

Ans. 201062400 square miles

PROBLEM II.

To find the solidity of a globe. *Rule*—Multiply the cube of the diameter by the decimal ,5236, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a globe whose diameter is 2 feet 6 inches? Ans. 8,181 + cubic feet.

2. What is the solidity of a globe whose diameter is 3 feet 4 inches? Ans. 19,3926 — cubic feet.

3. How many cubic miles are contained in the solidity of the globe we inhabit, if its diameter be 8000 miles? Ans. 268083200000 cubic miles.

PROBLEM III.

The solidity of a globe given, to find the diameter. *Rule*—Divide the solidity by ,5236, and extract the cube root of the quotient.

EXAMPLES.

1. The solidity of a globe is 2000 solid inches—What is its diameter? Ans. 15,618 + inches.

2. The solidity of a globe is 10 solid feet—What is its diameter? Ans. 2,67 + feet.

3. The solidity of a globe is 8 solid feet—What is its circumference?

Ans. 7,7943096 feet the circumference.

PROBLEM IV.

The convex surface of a globe given, to find its diameter. *Rule*—Divide the given area by 3,1416, and the square root of the quotient will be the diameter.

EXAMPLES.

1. What is the diameter of that globe, the area of whose convex surface is 14 square feet?

Ans. 2,1126 + feet.

2. The convex surface of a sphere is one square rood—What is its diameter. Ans. 3,5682 + rods.

3. The expense of gilding a ball at 1,80 per square foot, is 34 dollars—What is its diameter?

Ans. 2,452 + feet.

PROBLEM V.

To find the area of the convex segment of a globe, the diameter of the globe and the height of the segment given. *Rule*—Multiply the circumference of the whole sphere by the height of the segment, and the product will be the area of the convex surface.

EXAMPLES.

1. What is the area of the convex surface of a segment of a globe, the diameter of the globe being 6 feet, and the height of the segment 2 feet?

Ans. 37,6992 square feet.

2. The diameter of a globe is 8 feet, and the chord of the segment cut from it is three feet—What is the area of the convex surface of the segment?

Ans. 7,3387 + square feet.

3. The chord of the segment of a globe is 30 inches, and the height 6—What is the area of its convex surface? Ans. 5,7 + square feet.

PROBLEM VI.

To find the area of the convex surface of any zone of a globe. *Rule*—Multiply the circumference of the whole globe by the height or breadth of the zone, and the product will be the area required.

EXAMPLES.

1. What is the convex surface of a spherical zone whose breadth is 4 inches, and the diameter of the sphere from which it was cut 25 inches?

Ans. 314,16 square inches.

2. What is the area of a spherical zone cut from a globe of 30 inches diameter, if the diameter of the zone on one side be 16 inches and the other 10?

Ans. 17,4751 + feet.

3. The diameter of a globe is 40 inches, and if two segments be cut from it, the chord of the one 16 inches, and the height of the other segment 10 inches, what is the convex surface of the zone.

Ans. 24,72 square feet.

PROBLEM VII.

To find the solidity of a spherical segment, the diameter and height given. *Rule*—To three times the square of the semidiameter of the segment add the square of its height, and multiply the sum by the height of the segment, and that product by ,5236, the last product will be the solidity of the segment.

EXAMPLES.

1. The diameter of a spherical segment is 30 inches, and the height 8 inches—What is the solidity of the segment? Ans. 1,7913 + cubic feet.

2. The height of a spherical segment is 10 inches, and the chord or diameter 40—What is the solidity? Ans. 3,93912 cubic feet.

3. The diameter of a bowl is 20 inches, and the depth 6—How many gallons of beer will it hold? Ans. 3,7432 — beer gallons.

4. The diameter of a sphere is 18 inches—What is the solidity of a segment cut from it, the height being 3 inches? Ans. 226,1952 cubic inches.

5. Required the solidity of a spherical segment, the diameter of which is 20 inches, and the globe from which it was taken 30 inches.

Ans. 629,2326 + cubic inches the solidity required.

PROBLEM VIII.

To find the solidity of a frustrum or zone of a sphere. *Rule*—Add together the squares of the radii or semidiameter of the two ends and ⅓ the square of their distance, and multiply the sum by 3 times this distance, and the product by ,5236; the last product will be the solidity of the zone.

EXAMPLES.

1. If the diameter of one end of a spherical zone be 24 inches, and the diameter of the other 20, and the

distance of the two ends 9 inches, what is the solidity of the zone? Ans. 3831,1812 cubic inches.

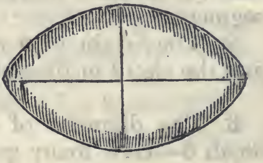
2. If from a globe 5 feet in diameter a segment be cut off 36 inches diameter, and another from the opposite side 18 inches in height, what is the solidity of the zone? Ans. 49,4802 cubic feet.

3. Required the solidity of the middle zone of a sphere whose top and bottom diameters are each 3 feet, and the breadth of the zone 4,2 feet.

Ans. 68,48 cubic feet.

DEFINITION.

A prolate spheroid is a solid generated by the revolution of a semi-ellipsis about the transverse diameter.



PROBLEM IX.

To find the solidity of a prolate spheroid. *Rule.*—Multiply the longest diameter by the square of the shortest, and that product by the decimal ,5236 and the product will be the solidity.

EXAMPLES.

1. The transverse diameter of a prolate spheroid is 50 inches, and the conjugate 40—What is its solidity? Ans. 24,2465 + cubic feet.

2. The two diameters of a prolate spheroid are 3 feet, and 2 feet—What is the solidity? Ans. 6,2832.

2. The two diameters of a prolate spheroid are 60 inches, and 40—What is the solidity?

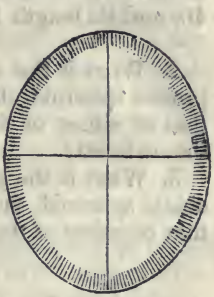
Ans. 29,0883 + cubic feet.

4. What is the solidity of a prolate spheroid whose diameters are 100, and 60 inches?

Ans. 109,083 + cubic feet.

DEFINITION.

An oblate spheroid is generated by the revolution of a semi-ellipsis about its conjugate diameter.



PROBLEM X.

To find the solidity of an oblate spheroid. *Rule*—Multiply the shortest diameter by the square of the longest, and that product by ,5236, the last product will be the solidity.

EXAMPLES.

1. The two diameters of an oblate spheroid are 100, and 60—Required the solidity.
Ans. 181,8 — cubic feet.

2. The transverse and conjugate diameters of an oblate spheroid are 40, and 30 inches—What is its solidity? Ans. 14,544 + cubic feet.

3. If the transverse diameter of an oblate spheroid be 20 inches, and the conjugate 15, what will be the solidity of the spheroid? Ans. 1,818 + cubic feet.

PROBLEM XI.

To find the solid content of the middle frustrum of a spheroid, its length, the middle diameter, and that of either of the ends being given:

I.—When the ends are circular or parallel to the revolving axis.

Rule—To twice the square of the middle diameter add the square of the diameter of either of the ends, and this sum multiplied by the length of the frustrum, and that product by ,2618, will give the solidity.

EXAMPLES.

1. In the middle frustrum of a spheroid the middle

diameter is 50 inches, and that of either of the ends is 40, and its length 18 inches—Required the solidity.
Ans. 18 — cubic feet.

2. What is the solidity of the middle frustrum of a prolate spheroid, the middle diameter being 60 inches, that of either of the two ends 36, and the distance of the ends 80? Ans. 102,9746 + cubic feet.

3. What is the solidity of the middle frustrum of an oblate spheroid, the middle diameter being 100 inches, that of either of the ends 80, and their distance 36?
Ans. 143,99 cubic feet.

PROBLEM XII.

II.—When the ends are elliptical or perpendicular to the revolving axis.

Rule—Multiply twice the transverse diameter of the middle section by its conjugate diameter, and to this product add the product of the transverse and conjugate diameters of either of the ends; multiply the sum thus found by the distance of the ends, or the height of the frustrum, and that product by ,2618 and it will give the solidity required.

EXAMPLES.

1. In the middle frustrum of an oblate spheroid the diameters of the middle section are 50, and 30 inches, those of the end 40, and 24, and its height 18—What is its solidity? Ans. 10,8 — cubic feet.

2. In the middle frustrum of a prolate spheroid, the diameters of the middle section are 100, and 60 inches, those of the end 80, and 48, and the length 36—What is its solidity? Ans. 53,669 cubic feet.

3. In the middle frustrum of an oblate spheroid, the diameters of the middle section are 100, and 60 inches, those of the end 60 and 36, and the length 80—What is the solidity of the frustrum. Ans. 171,6244 +.

PROBLEM XIII.

To find the solidity of the segment of a spheroid.

III.—When the base is parallel to the revolving axis.

Rule—Divide the square of the revolving axis by the square of the fixed axis, and multiply the quotient by the difference between three times the fixed axis and twice the height of the segment; multiply this product by the square of the height of the segment, and this product by ,5236, and it will give the solidity required.

EXAMPLES.

1. If the transverse axis of a prolate spheroid be 100, and the conjugate 60, and the height of the segment 10—Required the solidity. Ans. 5277,888 the solidity.

2. The axes of a prolate spheroid are 50 and 30—What is the solidity of that segment whose height is 5 and its base perpendicular to the fixed axis?
Ans. 659,736 the solidity.

3. If the diameters of an oblate spheroid be 100, and 50, what is the solidity of that segment whose height is 12, and its base perpendicular to the fixed axis?
Ans. 5202,4896 the solidity.

PROBLEM XIV.

IV.—When the base is perpendicular to the revolving axis.

Rule—Divide the fixed by the revolving axis, and multiply the quotient by the difference between three times the revolving axis and twice the height of the segment; and multiply the product thus found by the square of the height of the segment, and this product by ,5236, and it will give the required solidity.

EXAMPLES.

1. In the prolate spheroid the transverse axis is 100, the conjugate 50, and the height of the segment 12—What is the solidity? Ans. 19000,3968 the solidity.

2. Required the solidity of the segment of a prolate spheroid, its height being 6 inches, and the axes 50, and 30. Ans. 2450,448 the solidity.

PROBLEM XV.

To find the solidity of a parabolic conoid. *Rule*—Multiply the area of the base by half the altitude, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a paraboloid whose height is 84, and the diameter of its circular base 48?

Ans. 76001,5872 the solidity required.

2. What is the solidity of a paraboloid whose height is 50, and the diameter of its base 100?

Ans. 196350 the solidity.

3. What is the solidity of a parabolic conoid whose height is 30, and the diameter of the base 100?

Ans. 117810 the solidity.

4. Required the solidity of a parabolic conoid whose height is 30, and diameter 40?

Ans. 18849,6 the solidity.

PROBLEM XVI.

To find the solidity of the frustrum of a paraboloid, when its ends are perpendicular to the axis of the solid. *Rule*—Multiply the sum of the squares of the diameters of the two ends by the height of the frustrum, and the product by ,3927 and it will give the solidity.

EXAMPLES.

1. Required the solidity of the parabolic frustrum, the diameter of the greater end being 58, that of the less end 30, and the height 18.

Ans. 30142,4104 solidity required.

2. What is the solidity of the frustrum of a parabolic conoid, the diameter of the greater end being 60, that of the less 50, and the distance of the ends 10?

Ans. 23954,7 solidity required.

PROBLEM XVII.

To find the solidity of a hyperboloid. *Rule*—To the square of the radius of the base add the square of the middle diameter between the base and the vertex, and this sum multiplied by the altitude, and that product by ,5236, the last product will give the solidity.

EXAMPLES.

1. In the hyperboloid the altitude is 10, the radius or semidiameter of the base 12, and the middle diameter 15,8745—What is the solidity? Ans. 2073,454691.

2. In a hyperboloid the altitude is 50, the radius of the base 52, and the middle diameter 68—What is the solidity? Ans. 191847,04 the solidity.

PROBLEM XVIII.

To find the solidity of the frustrum of a hyperbolic conoid. *Rule*—Add together the squares of the greatest and least semidiameters, and the square of the whole diameter in the middle; multiply the sum by the altitude, and that product by ,5236, the last product will be the solidity.

EXAMPLES.

1. In the hyperbolic frustrum the length is 20, the diameter of the greater end 32; that of the less 24, and the middle diameter 28,1708—Required the solidity.
Ans. 12499,31532 the solidity.

2. What is the solidity of the frustrum of any hyperbolic conoid, whose greater diameter is 96, less diameter 54, the middle diameter 76,4264392, and the altitude 25? Ans. 116156,57 + the solidity.

3. The height of the frustrum of a hyperbolic conoid is 12 inches, the greatest and least diameters 10, and 6, and the middle diameter 8,5—Required the solidity.
Ans. 667,59 the solidity.

4. Required the content of the segment of any spindle, its length being 10, the greatest diameter 8, and the middle diameter 6.
Ans. 272,272 the solidity.

SECTION VII.

MENSURATION OF SOLIDS.

PROBLEM I.

The solidity of a cubic box is given, to find the length of the side. *Rule*—Extract the cube root of the solidity.

EXAMPLES.

1. The solidity of a cubic box is 5 bushels—What is the length of the side? Ans. 22,07.

2. The solidity of a cubic box is 282 solid inches—What is the length of its side? Ans. 6,5576.

PROBLEM II.

The side of a cubic box is given, to find the length of a diagonal extending from the lower corner to its opposite on the upper side. *Rule*—Extract the square root of the sum of the squares of the length, breadth, and thickness.

EXAMPLES.

1. What is the length of the diagonal drawn from one of the lower corners to its opposite upper corner of a cubic box, whose side is 20 inches?

Ans. 34,641 +.

2. The solidity of a cube is 3834 cubic inches—What is the length of a diagonal line reaching from one of the lower corners to its opposite upper corner?

Ans. 27,1088 + inches.

PROBLEM III.

The length, breadth, and solidity of a parallelopipedon are given, to find its thickness. *Rule*—Multiply the length by the breadth for a divisor, divide the solidity by it, the quotient will be the thickness.

EXAMPLES.

1. The length of a parallelopipedon is 40 feet, its breadth 18 inches, and solidity 80 feet—What is its thickness? Ans. 20 inches.

2. The length of a parallelopipedon is 20, the solidity 36 feet, and thickness 14 inches—What is its breadth? Ans. 18,51425 inches the breadth required.

PROBLEM IV.

The breadth, thickness, and solidity given, to find its length. *Rule*--Divide the solidity by the product of the breadth and thickness, and the quotient will be its length.

EXAMPLES.

1. The breadth and thickness of a parallelopipedon are each 20 inches, and the solidity 40 feet—What is its length? Ans. 14,4 feet.

2. How much in length that is 18 inches by 14, will make 30 solid feet? Ans. 17,1428 +

PROBLEM V.

The diameter and solidity of a cylinder given, to find its length. *Rule.*—Multiply the square of the diameter by ,7854 for a divisor, and divide the solidity by it, the quotient will be its length.

EXAMPLES.

1. The diameter of a cylinder is 20 inches—How much of its length will make 20 solid feet? Ans. 9,1673 + feet.

2. The diameter of a cistern in the form of a cylinder is 5 feet 6 inches—What must be its depth to hold 20 hogsheads? Ans. 7,09 — feet the depth.

3. How much in length of a round stick of timber

15 inches diameter, and 30 feet long, must I cut off to have 24 solid feet left? Ans. 10,443 + feet.

PROBLEM VI.

The length and solidity of a cylinder given, to find the diameter. *Rule*—Multiply the length by ,7854, and divide the solidity by the product, the square root of the quotient will be the diameter.

EXAMPLES.

1. The length of a cylinder is 30 feet, and solidity 60—What is its diameter?
Ans. 19,26 — inches the diameter.

2. The depth of a bushel measure is 8 inches—What must be its diameter? Ans. 18,5 inches.

3. The depth of a half bushel measure is 7,5 inches —What must be its diameter?
Ans. 13,1926 + inches.

PROBLEM VII.

The diameter and length of a cylinder are given, to find the number of solid feet left when hewn square. *Rule*—Multiply twice the square of the semi-diameter by the length, and the product will be the solidity.

EXAMPLES.

1. The diameter of a cylinder is 24 inches, and the length 25 feet—What will be its solidity when hewn square? Ans. 50 cubic feet.

2. The length of a cylinder is 40 feet, and the diameter 20 inches—What is its solidity when hewn square; and likewise the solidity of the chips that reduce it to a square?
Ans. 55,5555 + cubic feet the solidity, and 31,7111 solidity of the chips.

3. If a cylinder be 18 inches diameter, and 30 feet long, how much of its length will make 20 solid feet when hewn square? Ans. 21 feet 4 inches.

PROBLEM VIII.

The altitude of a cone and its solidity are given, to find its diameter. *Rule*—Multiply ,7854 by ⅓ of the altitude for a divisor, divide the solidity by it, and the quotient will be the square of the diameter, the square root of which will be the diameter.

EXAMPLES.

1. The altitude of a cone is 15 feet, and its solidity 30 feet—What is its diameter?

Ans. 33,1647 + inches the diameter.

2. The altitude of a cone is 9 feet, and its solidity 16 —What is its diameter? Ans. 31,27 — inches.

3. The solidity of a cone is 18 feet, its altitude 8— What is the diameter?

Ans. 35,18 — inches the diameter.

PROBLEM IX.

The solidity and diameter given to find the altitude. *Rule*—Multiply the square of the diameter by ,7854 for a divisor, divide the solidity by it, and the quotient will be ⅓ of the altitude.

EXAMPLES.

1. The diameter of a cone is 20 inches—What must be its altitude to make 20 solid feet?

Ans. 27,5019 + feet.

2. The content of a cone is 30 solid feet, its diameter is two feet—What is its altitude? Ans. 28,6476 feet.

PROBLEM X.

The altitude of a cone being given, to divide into two or more parts by sections parallel to the base, to find the perpendicular height of each part. *Rule*— Multiply the cube of the altitude by the numerator of the proportion left at the vertex, and divide the product by the denominator, the cube root of the quotient will be the altitude of the cone left at the vertex.

EXAMPLES.

1. The altitude of a cone is 6 feet, to be divided into two equal parts by a line parallel to the base—Required the perpendicular height of each part.

Ans. 4,7622 feet, and 1,2378 the thickest section.

2. The altitude of a cone is 10 feet, to be divided into three equal parts by sections parallel to the base—Required the perpendicular height of each part.

Ans. 6,9336, 1,8022, and 1,2642 feet.

3. The altitude of a cone is 12 feet, to be divided into three parts in the proportion of 3, 4, 5, by sections parallel to the base—Required the perpendicular height of each part.

Ans. 7,5595 +, 2,4671 +, and 1,9734 + feet the altitudes.

PROBLEM XI.

The base and altitude of a cone are given, to find what length of wire may without loss of metal be drawn from it, the diameter of the wire also given. *Rule*—Multiply the square of the base of the cone by $\frac{1}{3}$ of its altitude for a dividend; square the diameter of the wire for a divisor; the quotient resulting from the division will be the length.

EXAMPLES.

1. What length of a wire $\frac{1}{10}$ of an inch diameter can be drawn from a cone whose altitude is 6 inches, and diameter 4? Ans. 88,888 + yards.

2. The base of a conical ingot of gold is 3 inches, and altitude 9 inches—What length of wire may be drawn from it, the diameter of the wire being the one thousandth part of an inch? Ans. $426\frac{3}{22}$ miles.

3. The altitude of a cone is 12 inches, and its base 4 inches in diameter—What length of wire, without loss of metal, may be drawn from it; the diameter of the wire being the $\frac{1}{8}$ of an inch? Ans. 256 feet.

SECTION VIII.

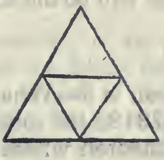
OF REGULAR BODIES.

A Regular body is a solid contained under a certain number of similar and equal plane figures. The whole number of regular bodies which can be formed is five.

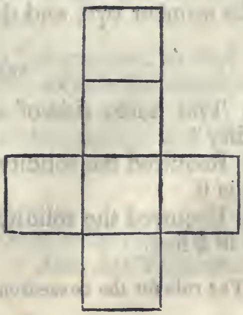
1. The Tetraedron or equilateral triangular pyramid, which has four equilateral triangular faces.
2. The Hexaedron or cube, which has six square faces.
3. The Octaedron, which has eight equilateral triangular faces.
4. The Dodecahedron, which has twelve equal pentagonal faces.
5. The Icosaedron, which has 20 equilateral triangular faces.

If the following figures be made of pasteboard, and the lines be cut half through so that the parts may be folded together, they will represent the five regular bodies above mentioned.

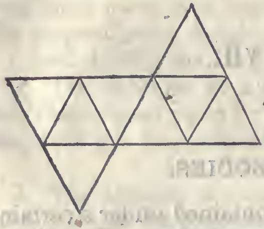
Tetraedron.



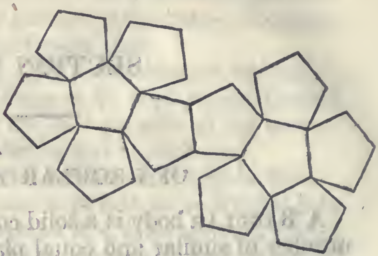
Hexaedron.



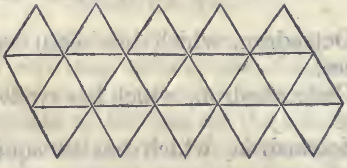
Octaedron.



Dodecahedron.



Icosaedron.



PROBLEM I.

To find the solidity of the tetraedron. *Rule**—Multiply $\frac{1}{12}$ of the cube of the linear side by the square root of the number two, and the product will be the solidity.

EXAMPLES.

1. The linear side of a tetraedron is 5—What is its solidity? Ans. 14,73125 the solidity.

2. Required the solidity of a tetraedron whose linear side is 6. Ans. 25,4556.

3. Required the solidity of a tetraedron whose linear side is 4 feet. Ans. 7,5424 cubic feet.

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* The rule for the hexaedron or cube, has been given before.

PROBLEM II.

The solidity of a tetraedon being given, to find the side. *Rule*—Divide the solidity by the square root of the number two, multiply the quotient by 12, and the cube root of the product will be the length of the linear side.

EXAMPLES.

1. The solidity of a tetraedon is 9,46 feet—Required the length of the linear side. Ans. 4,314 — feet.

2. The solidity of a tetraedon or equilateral triangular pyramid is 25,452 feet—What is the length of the linear side? Ans. 6 feet.

3. Required the length of the linear side of an equilateral triangular pyramid, whose solidity is 36 feet. Ans. 6,734 +.

PROBLEM III.

To find the solidity of an octaedron. *Rule*—Multiply $\frac{1}{3}$ of the cube of the linear side by the square root of the number two, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of an octaedron, whose linear side is 6 feet? Ans. 101,8224 cubic feet.

2. Required the solidity of an octaedron, whose linear side is 5 inches. Ans. 58,925 cubic inches.

3. What is the solidity of an octaedron whose linear side is 8 feet? Ans. 241,3568 cubic feet.

PROBLEM IV.

The solidity of an octaedron being given, to find the length of the linear side. *Rule*—Divide the solidity by the square root of the number two, and multiply the quotient by 3, the cube root of the product will be the length of the linear side.

EXAMPLES.

1. The solidity of an octaedron is 32 feet—What is the length of the side? Ans. 4,0793 + feet.

2. Required the length of the side of an octaedron, whose solidity is 300 feet. Ans. 8,6 feet.

3. The solidity of an octaedron is 60 feet—What is the length of the linear side? Ans. 5,032 + feet.

PROBLEM V.

To find the solidity of the dodecahedron. *Rule*—Multiply five times the cube of the linear side by 1,52362, and the product will be the solidity.

EXAMPLES.

1. The linear side of the dodecahedron is 4 feet—What is its solidity? Ans. 490,4384 cubic feet.

2. What is the solidity of a dodecahedron whose linear side is 6 inches? Ans. 1655,2296 cubic inches.

3. Required the solidity of a dodecahedron, whose linear side is two feet. Ans. 61,3048 cubic feet.

PROBLEM VI.

The solidity of a dodecahedron being given, to find the length of the linear side. *Rule*—Multiply 1,53262 by five for a divisor, and divide the solidity by it, the quotient will be the cube of the side required; the cube root of which will be the side.

EXAMPLES.

1. The solidity of a dodecahedron is 206,901 solid inches—What is the length of the linear side?
Ans. 3 inches.

2. The solidity of a dodecahedron is 600 feet—What is the length of the linear side? Ans. 4,278 + feet.

3. Required the length of the side of a dodecahedron, whose solidity is 7,6631 feet. Ans. 1 foot.

PROBLEM VII.

To find the solidity of an icosaedron. *Rule*—To three times the square root of the number five add 7, and divide the sum by two, then the square root of this quotient being multiplied by $\frac{5}{6}$ of the cube of the linear side, will give the solidity required.

EXAMPLES.

1. If the linear side of the icosaedron be 3, what is the solidity? Ans. 58,905675 the solidity required.

2. What is the solidity of an icosaedron whose linear side is 4 feet? Ans. 139,62826 cubic feet.

3. Required the solidity of an icosaedron, whose linear side is 1. Ans. 2,181691 the solidity required.

PROBLEM VIII.

The solidity of an icosaedron being given, to find the length of the linear side. *Rule*—To three times the square root of the number five add 7, and divide the sum by two, and extract the square root of the quotient; divide the solidity by $\frac{5}{6}$ of that root, the quotient thence arising will be the cube of the linear side, the cube root of which will be the side required.

EXAMPLES.

1. The solidity of an icosaedron is 58,905675 inches—What is the length of the linear side?
Ans. 3 inches.

2. The solidity of an icosaedron is 600 feet—What is the length of the linear side? Ans. 6,503.

3. Required the length of the linear side of an icosaedron, whose solidity is 2,181691 feet. Ans. 1.

SECTION IX.

MENSURATION OF REGULAR BODIES.

TABLE OF SURFACES AND SOLIDS OF THE FIVE REGULAR BODIES, THE SIDE BEING UNITY, OR ONE.

No. of sides.	Names.	Areas of Surfaces.	Solidity.
4	Tetraedron	1,73205	0,11785
6	Hexaedron	6,00000	1,00000
8	Octaedron	3,46410	0,47140
12	Dodecahedron	20,64578	7,66312
20	Icosaedron	8,66025	2,18169

PROBLEM I.

The superficies and solidity of any of the five regular bodies may be found as follows.

Rule 1.—Multiply the tabular area by the square of the length of the given side, the product will be the area of the surface.

Rule 2.—Multiply the tabular solidity by the cube of the given side, and the product will be the solidity.

EXAMPLES.

1. The side of a tetraedron is 4—What is the area of its surface? Ans. 27,7128 the area.

2. The side of a tetraedron is 6 feet—Required the area of its surface. Ans. 62,3538 square feet.

3. The side of a tetraedron is 8 inches—What is the area of its surface? Ans. 110,8512 square inches.

PROBLEM II.

The area of the surface of a tetraedron being given, to find the length of the side. *Rule*—Divide the given area by 1,73205, and the square root of the quotient will be the length of the linear side.

EXAMPLES.

1. The area of the surface of a tetraedron is 8 feet—What is the length of the side? Ans. 2,1625 + feet.

2. The area of the surface of a tetraedron is 12 feet —What is the length of its side? Ans. 2,632 + feet.

3. Required the length of the side of a tetraedron, the area of whose surface is 20 inches.

Ans. 3,398 + inches.

PROBLEM III.

The side of a Hexagon being given, to find the area of its surface. *Rule*—Multiply the square of the given side by 6, and the product will be the area of its surface.

EXAMPLES.

1. The side of a hexagon is 8 inches—What is the area of its surface? Ans. 384 square inches.

2. The area of a hexagon is 10 inches—What is the area of its surface? Ans. 600 square inches.

PROBLEM IV.

The area of the surface of a hexagon being given, tc find the length of the side. *Rule*—Divide the area of the surface by 6, and the square root of the quotient will be the length of the side.

EXAMPLES.

1. The area of the surface of a hexagon is 216 feet —What is the length of the side? Ans. 6 feet.

2. Required the length of the side of a hexagon, the area of whose surface is 300 feet. Ans. 7,071 +.

PROBLEM V.

The length of the side of an octaedron being given, to find the area of its surface. *Rule*—Multiply the square of the given side by 3,4641, and the product will be the area of the surface.

EXAMPLES.

1. The length of the side of an octaedron is 3 feet —What is the area of its surface?

Ans. 31,1769 square feet.

2. Required the area of the surface of an octaedron, whose side is 4 feet 6 inches.

Ans. 70,148 + square feet.

3. What is the area of the surface of an octaedron, whose side is 6 feet in length?

Ans. 124,7076 square feet.

PROBLEM VI.

The area of the surface of an octaedron being given, to find the length of the side. *Rule*—Divide the given area by 3,4641, and the square root of the quotient will be the length of the side.

EXAMPLES.

1. The area of an octaedron is 20 square feet—What is the length of the side? Ans. 2,4 feet.

2. What is the length of the side of an octaedron, the area of whose surface is 36 square feet?

Ans. 3,2237 + feet.

3. Required the length of the side of an octaedron, the area of whose surface is 24 square feet.

Ans. 2,6321 + feet.

PROBLEM VII.

The length of the side of a dodecahedron being given, to find the area of its surface. *Rule*—Multiply the square of the given side by 20,64578, and the product will be the area of its surface.

EXAMPLES.

1. The length of the side of a dodecahedron is 6 feet —What is the area of its surface?

Ans. 743,24808 square feet.

2. Required the area of the surface of a dodecahedron, the length of whose side is 5½ feet.

Ans. 624,534845 square feet.

3. What is the area of the surface of a dodecahedron, the length of whose side is 3 feet?

Ans. 185,812 square feet.

PROBLEM VIII.

The area of the surface of a dodecahedron being given, to find the length of the side. *Rule*—Divide the given area by 20,64578, and the square root of the quotient will be the length of the side required.

EXAMPLES.

1. The area of the surface of a dodecahedron is 840 feet—What is the length of its side?

Ans. 6,3785 + feet.

2. Required the length of the side of a dodecahedron, the area of whose surface is 150 square yards.

Ans. 2,6954 + yards.

3. What is the length of the side of a dodecahedron, the area of whose surface is 120 square feet?

Ans. 2,4108 + feet.

PROBLEM IX.

The length of the side of an icosaedron being given, to find the area of its surface. *Rule*—Multiply the square of the given side by 8,66025, and the product will be the area of its surface.

EXAMPLES.

1. What is the area of the surface of an icosaedron, the length of whose side is 6 feet?

Ans. 311,769 square feet.

2. Required the area of the surface of an icosaedron, whose side is 4½ feet. Ans. 175,37 square feet.

3. If the side of an icosaedron is 3 feet, what is the area of its surface? Ans. 77,94225 square feet.

PROBLEM X.

The area of the surface of an icosaedron being given, to find the length of the side. *Rule*—Divide the given area by 8,66025, and the square root of the quotient will be the length of the side required.

EXAMPLES.

1. The area of the surface of a icosaedron is 294 square feet—What is the length of its side?

Ans. 5,8265 + feet.

2. Required the length of the side of an icosaedron, the area of whose surface is 16 square yards.

Ans. 1,36 — yards.

3. What is the length of the side of an icosaedron, the area of whose surface is 284 square feet?

Ans. 5,7265 + feet.

SECTION X.

MENSURATION OF REGULAR BODIES.

PROBLEM I.

The side of a tetraedron being given, to find the solidity. *Rule*—Multiply the cube of the given side by 0,11785, and the product will be the solidity.

EXAMPLES.

1. The side of a tetraedron is 7 feet—What is its solidity? Ans. 40,42255 cubic feet.

2. The side of a tetraedron is 3 feet 6 inches—What is its solidity? Ans. 5,0528 + cubic feet.

3. Required the solidity of a tetraedron, whose linear side is 5 feet. Ans. 14,73125 cubic feet.

PROBLEM II.

The solidity of a tetraedron being given, to find the length of the side. *Rule*—Divide the solidity by ,11785, and the cube root of the quotient will be the length of the side.

EXAMPLES.

1. The solidity of a tetraedron is 230 solid feet—What is the length of its side? Ans. 12,44 + feet.

2. An equilateral triangular pyramid contains 128 feet—What is the length of the side?
Ans. 10,279 + feet.

3. Required the length of the side of a tetraedron, whose solidity is 3 feet. Ans. 2,941 + feet.

PROBLEM III.

Having the diameter of a globe given, to find the perpendicular altitude of the largest tetraedron that can be formed within it. *Rule*—Multiply the diameter by three, and divide the product by 11; then subtract the quotient from the diameter or axis of the globe given, and the remainder will be the perpendicular altitude required.

EXAMPLES.

1. The diameter of a globe is 12 inches—What is the perpendicular altitude of the greatest tetraedron that can be formed within it? Ans. 8,7273 + inches.

2. If the axis of a globe be 5 feet, what is the perpendicular altitude of the greatest equilateral triangular pyramid that can be cut from it? Ans. 3,6364 feet.

PROBLEM IV.

Having the diameter of a globe given, to find the length of the side of the greatest tetraedron that can be made from it. *Rule*—Multiply the given diameter by three, and divide the product by eleven, subtract the quotient from the diameter, and multiply the remainder by the quotient, and that product by 4; the square root of the last product will be the side required.

EXAMPLES.

1. The axis of a globe is 30 inches—What is the length of the side of the greatest tetraedron that can be made from it? Ans. 26,72 inches.

2. What is the length of the side of the greatest tetraedron that can be made from a globe, whose diameter is 22 inches? Ans. 19,59 + inches.

3. I demand the length of the side of the greatest tetraedron that can be made from a globe whose diameter is 11 inches.

Ans. 9,798 — inches the length of the side.

PROBLEM V.

Having the length of the side of a tetraedron given, to find the diameter of the least globe from which it could have been taken. *Rule*—Suppose the diameter of the globe to be any convenient number, and find the solidity of the greatest tetraedron that can be made from it; find also the solidity of the tetraedron given in the question: then as the solidity of the pyramid cut from the supposed globe is to the cube of the diameter of the supposed globe, so is the solidity of the tetraedron to the cube of the diameter of the globe required, the cube root of which will be the diameter.

EXAMPLES.

1. The side of an equilateral triangular pyramid is 10 inches—What is the diameter of the smallest globe from which it could have been made?
Ans. 11,226 inches.

2. What is the diameter of the least globe from which a tetraedron may be made whose side is 6 inches?
Ans. 6,735 + inches.

3. If the side of an equilateral triangular pyramid be 24 inches, what is the diameter of the smallest globe from which it could have been made?
Ans. 26,91 + inches.

PROBLEM VI.

The length of the side of an octaedron being given, to find its solidity. *Rule*—Multiply the cube of the length of the side by ,4714, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of an octaedron whose side is 3 feet 3 inches? Ans. 16,1822 + cubic feet.

2. Required the solidity of an octaedron, whose side is 2 inches. Ans. 3,7712 cubic inches.

3. The side of an octaedron is 5 feet—What is its solidity? Ans. 58,925 cubic feet.

PROBLEM VII.

The solidity of an octaedron being given, to find the length of the side. *Rule*—Divide the solidity by ,4714, and the cube root of the quotient will be the length of the side required.

EXAMPLES.

1. The solidity of an octaedron is 12 feet—What is the length of the side? Ans. 2,9416 + feet.
2. Required the length of the side of an octaedron, whose solidity is 128 feet. Ans. 6,475 + feet.
3. What is the length of the side of an octaedron, whose solidity is 9 solid feet? Ans. 2,6727 + feet.

PROBLEM VIII.

The side of a dodecahedron being given, to find its solidity. *Rule*—Multiply the cube of the length of the side by 7,66312, and the product will be the solidity.

EXAMPLES.

1. The length of the side of the dodecahedron is 2 feet—What is its solidity? Ans. 61,30496 cubic feet.
2. What is the solidity of a dodecahedron, whose side is 3 feet in length? Ans. 206,90424 feet.
3. Required the solidity of a dodecahedron, whose side is 4 inches. Ans. 490,43968 cubic inches.

PROBLEM IX.

The solidity of a dodecahedron being given, to find the length of the side. *Rule*—Divide the solidity by 7,66312, and the cube root of the quotient will be the length of the side required.

EXAMPLES.

1. What is the length of the side of a dodecahedron, whose solidity is 500 solid feet? Ans. 4,026 — feet.
2. Required the length of the side of a dodecahedron whose solidity is 128 feet? Ans. 2,556 + feet.

3. If the solidity of a dodecahedron be 1728 feet, what is the length of the side? Ans. 6,0866 + feet.

PROBLEM X.

The length of the side of an icosaedron being given, to find its solidity. *Rule*—Multiply the cube of the side by 2,18169, and the product will be the solidity.

EXAMPLES.

1. The length of the side of an icosaedron is 3 feet—What is its solidity? Ans. 58,90563 cubic feet.
2. The length of the side of an icosaedron is 4 feet—What is its solidity? Ans. 139,62816 cubic feet.
3. Required the solidity of an icosaedron, the length of whose side is 6 feet. Ans. 471,245 + cubic feet.

PROBLEM XI.

The solidity of an icosaedron being given, to find the length of the side. *Rule*—Divide the solidity by 2,18169, and the cube root of the quotient will be the length of the side required.

EXAMPLES.

1. The solidity of an icosaedron is 24 solid feet—What is the length of its side? Ans. 2,22398 + feet.
2. What is the length of the side of an icosaedron, whose solidity is 128 feet? Ans. 3,841 + feet.
3. Required the length of the side of an icosaedron, whose solidity is 16 feet. Ans. 1,9428 + feet.

SECTION XI.

MENSURATION OF SOLIDS.

PROBLEM I.

The diameter of a globe being given, to find the length of the side of a tetraedron containing the same solidity. *Rule*—Multiply the cube of the diameter by ,5236, and divide the product by ,11785, and the cube root of the quotient will be the length of the side.

EXAMPLES.

1. The diameter of a globe is 8 inches—What is the length of the side of a tetraedron that contains the same solidity? Ans. 13,1516 + inches.

2. Required the length of the side of a tetraedron, whose solidity is equal to the solidity of a globe whose diameter is 6 feet. Ans. 9,8637 +.

PROBLEM II.

The length of the side of an octaedron being given, to find the length of the side of a tetraedron that shall contain the same solidity. *Rule*—Multiply the cube of the given side by ,4714, and divide the product by ,11785, and the cube root of the quotient will be the side required.

EXAMPLES.

1. The length of the side of an octaedron is 20 inches—What is the length of the side of a tetraedron that contains the same solidity? Ans. 31,748 + inches.

2. The length of the side of an octaedron is 3 feet—Required the length of the side of a tetraedron that contains the same solidity. Ans. 4,7622 + feet.

PROBLEM III.

The side of a dodecahedron being given, to find the side of an octaedron that contains the same solidity. *Rule*—Multiply the cube of the side of the dodecahedron by 7,66312, and divide the product by ,4714, and the cube root of the quotient will be the length of the required side.

EXAMPLES.

1. The side of a dodecahedron is 4 feet—What is the length of the side of an octaedron that contains the same solidity? Ans. 10,1328 + feet.

2. The length of the side of a dodecahedron is 6 feet —Required the length of the side of an octahedron that contains the same solidity. Ans. 15,2 — feet.

PROBLEM IV.

The length of the side of an icosaedron being given, to find the length of the side of a dodecahedron that contains the same solidity. *Rule*—Multiply the cube of the length of the side of the icosaedron by 2,18169, and divide the product by 7,66312, and extract the cube root of the quotient.

EXAMPLES.

1. The length of the side of an icosaedron is 12 feet —What is the length of the side of a dodecahedron that contains the same solidity? Ans. 7,89 + feet.

2. The length of the side of an icosaedron is 10 feet —What is the length of the side of a dodecahedron that contains the same solidity? Ans. 6,5985 + feet.

SECTION XII.

MENSURATION OF RINGS.

PROBLEM I.

To find the convex superficies of a cylindric ring. *Rule*—To the thickness of the ring, add the inner diameter, and multiply the sum by the thickness, and the product by 9,8696, will give the area of the surface required.

EXAMPLES.

1. The thickness of a cylindric ring is 3 inches, and the inner diameter 12 inches—What is its convex superficies? Ans. 444,132 square inches the area.

2. The thickness of a cylindric ring is 4 inches, and the inner diameter 18 inches—What is the area of the convex surface? Ans. 868,5248 inches.

3. The inner diameter of a cylindric ring is 18 inches, and the thickness 2 inches—What is the area of the convex surface? Ans. 394,784 inches.

PROBLEM II.

To find the solidity of a cylindric ring. *Rule*—To the thickness of the ring add the inner diameter, and this sum being multiplied by the square of half the thickness, and that product again by 9,8696 will give the solidity.

EXAMPLES.

1. What is the solidity of an anchor ring, whose inner diameter is 9 inches, and the thickness of metal 3 inches? Ans. 266,4792 cubic inches.

2. The inner diameter of a cylindric ring is 12 inches, and its thickness 4 inches—What is its solidity?
Ans. 631,6544 cubic inches.

3. Required the solidity of a cylindric ring, whose thickness is 2 inches, and its inner diameter 16 inches.
Ans. 197,6528 cubic inches.

4. I demand the solidity of a cylindric ring, whose inner diameter is 12 inches, and thickness five inches.
Ans. 1048,645 cubic inches the solidity.

PROBLEM III.

The solidity and thickness of a cylindric ring being given, to find the inner diameter. *Rule*—Divide the solidity by 9,8696, and that quotient by the square of half the thickness, from which subtract the thickness, and the remainder will be the inner diameter of the ring.

EXAMPLES.

1. The thickness of a cylindric ring is 4 inches, and its solidity 789,568 solid inches—What is its inner diameter? Ans. 16 inches.

2. Required the inner diameter of a cylindric ring, whose solidity is 138,1744 inches, and thickness 2 inches. Ans. 12 inches.

3. What is the inner diameter of a cylindric ring, whose solidity is one solid foot, and thickness 4 inches?
Ans. 39,77.

4. If the solidity of a cylindric ring be 4 solid feet, and the thickness 3,5 inches, what is the inner diameter? Ans. 18,765 feet.

5. What must be the inner diameter of a cylindric ring, whose solidity is 1 solid inch, and thickness $\frac{1}{8}$ of an inch? Ans. 25,8132 + inches.

SECTION XIII.

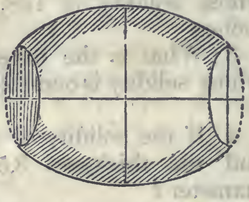
MEASURING CASKS.

THERE are four different forms of casks, which are frustrums of different kinds of solids, named from the greater or less apparent curvation of their sides.

1. The middle frustrum of a prolate spheroid.
2. The middle frustrum of a parabolic spindle.
3. The two equal frustrums of a paraboloid.
4. The two equal frustrums of a cone.

PROBLEM I.

To find the solidity of a cask of the first form. *Rule*—To the square of the head diameter add twice the square of the diameter at the bung, and multiply the sum by the length of the cask; divide the product by 1077 for ale gallons or 882 for wine gallons.



EXAMPLES.

1. Required in wine gallons the content of a spheroidal cask, whose length is 40 inches, bung diameter 32, and head diameters each 24.

Ans. 119 wine gallons.

2. What is the solid content of a spheroidal cask, whose length is 45 inches, bung diameter 34 inches, and head diameters each 25, in ale gallons?

Ans. 122,716 — ale gallons.

3. A spheroidal cask is 42 inches long, and the

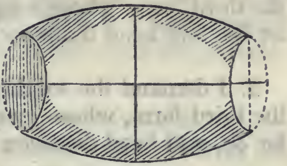
diameter at the bung 32 inches, and the head diameters each 23 inches—What is its content in wine gallons?

Ans. 122,7147 — wine gallons.

4. I demand the solidity in ale gallons of a spheroidal cask, whose length is 50 inches, bung diameter 40, and the diameter of each head 32 inches.

Ans. 196,1 ale gallons.

PROBLEM II.



To find the solidity of a cask of the second form, namely, the middle frustrum of a parabolic spindle. *Rule*—To the square of the head diameter add double the square of the bung diameter, and from the sum take four-tenths of the square of the difference of the diameters, and multiply the remainder by the length, and divide the product by 1077 for ale gallons, or by 882 for wine gallons.

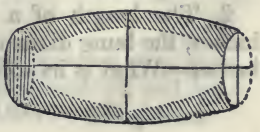
EXAMPLES.

1. What is the solidity in wine gallons, of a cask whose length is 40, bung diameter 31, and head diameters 24 inches? Ans. 112,4 wine gallons.

2. Required the solidity in ale gallons, of a cask of the second form, being that of the middle frustrum of a parabolic spindle, whose dimensions are, length 42 inches, bung diameter 32, and head diameters 24 inches. Ans. 101,33 ale gallons.

3. Required the number of wine gallons, in a cask of the form of the middle frustrum of a parabolic spindle, whose length is 45 inches, bung diameter 32, and head diameters 25 inches. Ans. 135,3775 + wine gallons.

PROBLEM III.



To find the solidity of a cask of the third form. *Rule*—To the square of the bung diameter add the square of the head diameter; multiply the sum by the length, and that product by ,0014 for ale gallons, or by ,0017 for wine gallons.

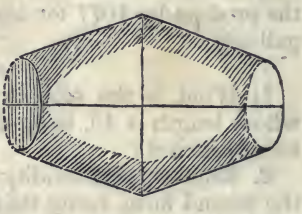
EXAMPLES.

1. What is the solidity in wine gallons, of a cask of the form of two equal frustrums of a paraboloid, whose length is 40 inches, bung diameter 32 inches, and head diameter 24 inches? Ans. 108,8 wine gallons.

2. Required the solidity in ale gallons, of a cask of the third form, whose length is 50 inches, bung diameter 30, and head diameter 20 inches.
Ans. 91 ale gallons.

3. I demand the solidity in wine gallons, of a cask of the third form, whose length is 60 inches, bung diameter 30, and head diameter 20 inches.
Ans. 132,6 wine gallons.

PROBLEM IV.



To find the solidity of a cask of the fourth form. *Rule*—Multiply the bung and head diameters together, and to that product add $\frac{1}{3}$ of the square of their difference, and multiply that sum by the length, and the product by ,7854, and divide the last product by 231 for wine gallons, or by 282 for ale gallons.

EXAMPLES.

1. The bung diameter of a cask, of the form two equal frustrums of a cone, is 32 inches, length 40, and the head diameter 24 inches—Required the solidity in wine gallons. Ans. 107,3494 — wine gallons.

2. The length of a cask of the fourth form is 50 inches, the bung diameter 30, and the head diameter 21 inches—What is its solidity in ale gallons?
Ans. 91,49 + ale gallons.

3. The head diameters of a cask of the fourth form are 18 inches, the bung diameter 30, and the length 50, inches—What are its contents in wine gallons?

Ans. 99,96 wine gallons.

PROBLEM V.

To find the content of a cask by four dimensions. *Rule*—Add together the squares of the bung and head diameters, and the square of double the diameter taken in the middle between the bung and head; then multiply the sum by the length of the cask, and that product by ,0004$\frac{2}{3}$ for ale gallons or by ,0005$\frac{2}{3}$ for wine gallons.

EXAMPLES.

1. Required the content in wine gallons, of any cask, whose length is 40, the bung diameter 32, the head diameter 24, and the middle diameter between the bung and head 28$\frac{3}{4}$ inches. Ans. 111,2 wine gallons.

2. What is the solidity in ale gallons, of a cask, whose length is 50, the bung diameter 30, the head diameter 20, and the middle diameter between the bung and head 26 inches? Ans. 93,425 + ale gallons.

3. What is the solidity in wine gallons of a cask, whose length is 20, bung diameter 16, the head diameter 12, and the diameter in the middle between them 14$\frac{1}{2}$ inches? Ans. 15,197 + wine gallons.

PROBLEM VI.

To find the content of any cask, having three dimensions only given. *Rule*—To 39 times the square of the bung diameter, add 25 times the square of the head diameter, and 26 times the product of those two diameters; then multiply the sum by the length, and that product by ,00034, and divide the last product by 9 for wine gallons, or by 11 for ale gallons.

EXAMPLES.

1. Required the solidity in wine gallons, of a cask, whose length is 40 inches, bung diameter 32, and head diameter 24 inches. Ans. 112,2816 wine gallons.

2. What is the content in ale gallons, of a cask, whose lengh is 50, bung diameter 36, and head diameters, 24 inches? Ans. 135,085 ale gallons.

3. The number of wine gallons are required, which a cask 48 inches long, bung diameter 30, and head diameters 20 inches is capable of containing.
Ans. 110,0693 + wine gallons.

OF THE ULLAGE OF CASKS.

The ullage of a cask is what it contains when only partly filled.

The cask is considered either standing on the end with its axis perpendicular to the horizon, or as lying on its side with its axis parallel to the horizon.

PROBLEM VII.

To find the ullage of a cask standing on the end. *Rule*—Add together the squares of the diameter at the surface of the liquor, the square of the diameter of the nearest end, and the square of twice the diameter in the middle between the other two; multiply the sum by $\frac{1}{6}$ of the distance between the surface and the nearest end, and that product by ,0028 for ale gallons, or ,0034 for wine gallons.

EXAMPLES.

1. If the diameter at the surface of the liquor in a standing cask be 32 inches, the diameter of the nearest end 24, the middle diameter 29, and the distance between the surface of the liquor and the nearest end 12, required in wine gallons, the quantity of liquor in said cask. Ans. 33¾ wine gallons.

2. The three diameters being 24, 27 and 29 inches —Required the ullage in ale gallons, the liquor being ten inches deep. Ans. 20,22 + ale gallons.

PROBLEM VIII.

To ullage a cask lying with its axis parallel to the horizon. *Rule*—Divide the number of inches of the depth of the liquor by the bung diameter; find the quotient in the column of versed sines in the table of circular segments, take out its corresponding area and multiply it by the whole content of the cask, and the product again by $\frac{5}{4}$, the product will be the ullage nearly.

EXAMPLES.

1. If the bung diameter of a cask lying horizontally be 32 inches, and its solidity 92 ale gallons, and the depth of the liquor 8 inches, what is its ullage?

Ans. 17,65779 ale gallons.

2. If the whole capacity of a lying cask be $49\frac{2}{3}$ wine gallons, the bung diameter 24 inches, and the distance from the bung to the surface of the liquor 6 inches, how many gallons are contained in the cask?

Ans. 40,134 wine gallons.

3. The whole capacity of a cask is 119 wine gallons, and the bung diameter 32 inches, and the distance from the bung to the surface of the liquor 12 inches—How many gallons are contained in the cask?

Ans. 79 wine gallons.

SECTION XIV.

ARTIFICERS' WORK.

Artificers compute the value of their work by different measures, viz.

1. Glazing and mason's flat work, &c. by the foot.
2. Painting, plastering, paving, &c. by the yard.
3. Flooring, partitioning, roofing, tiling, &c. by the square, each consisting of 100 square feet.
4. Brick and stone work by the perch of 16½ solid feet each.

The measures made use of in artificers' works are contained in the following table.

12	inches make . .	1 lineal foot,
144	square inches . .	1 square foot,
9	square feet . . .	1 square yard,
100	square feet . . .	1 square,
272¼	square feet . . .	1 square perch,
or 30¼	square yards . .	1 square perch,
16½	solid feet	1 solid perch.

Of Bricklayers' Work.

PROBLEM I.

Bricklayers compute their work at the rate of a brick and a half in thickness, and if a wall be more or less than this standard it must be reduced to it by the following. *Rule*—Multiply the superficial content of the wall in feet by the number of half bricks in the thickness, divide the product by three, and the quotient will be the content required.

EXAMPLES.

1. How many square perches are there in a wall 48 feet long, 12 feet 6 inches high, and 2½ bricks thick? Ans. 3,673 + square perches.

2. How many square perches are there in a wall 63 feet 3 inches long, 16 feet 9 inches high, and 2½ bricks thick? Ans. 6,452 + square perches.

3. If each side wall of a building be 60 feet long on the outside, each end wall 35 feet 6 inches in the inside, and the height of the building 30 feet, and the gable at each end of the wall 10 feet, what is the content in square perches, the whole being 2 bricks thick? Ans. 29,8 square perches.

PROBLEM II.

Of Masons' Work.

All sorts of stone work are performed by masons, and the measure made use of is the solid perch, containing 16½ solid feet.

EXAMPLES.

1. How many solid perches of stone are contained in a cellar wall, the length being 45 feet 6 inches on a side, and the breadth within 24 feet at each end, 6 feet 9 inches high, and 2 feet thick? Ans. 113,72 + perches.

2. What is the expense of making a stone wall under a building, whose length is 42 feet, breadth on the outside 26 feet, the height of the wall being 6 feet 6 inches, and 2 feet thick, at 40 cents per solid perch? Ans. $40,339 +.

3. A cellar wall is 36 feet in length on each side, the breadth in the inside 24 feet, the height 6 feet, and the thickness of the wall one foot 9 inches—What is the expense for stone and workmanship at ,90 cents per perch? Ans. $68,724 the expense.

4. The dimensions of a certain building are as follow, viz. 58 feet by 26 on the outside, height of the building

22 feet, height of the gable at each end 12 feet, thickness of the wall 15 inches; two doors 4½ by 8 feet each; 28 windows, each 3½ by 6 feet—What will the workmanship amount to at 56 cents a perch, making no deduction for doors and windows; and what the expense for stone at 44 cents a perch, deducting doors and windows?

Ans. $164,08 the workmanship, and $106,92 the value of the stone.

PROBLEM III.

Carpenters' and Joiners' Work.

The work of carpenters and joiners is that of flooring, partitioning, roofing, &c., and is measured by the square of 100 feet.

EXAMPLES.

1. If a floor be 60 feet long, 28 feet 9 inches broad, how many squares will it contain?

Ans. 17,25 squares.

2. What is the expense of flooring a building 45 feet 6 inches long, 26 feet 9 inches wide, two stories high, at 1,36 cents per square? Ans. $49,6587.

3. A partition is 96 feet 9 inches long, and 11 feet 6 inches broad—How many squares will it contain?

Ans. 11,12625 squares.

4. If the length of a building be 52 feet 6 inches, and the breadth 30 feet 3 inches, what will be the expense of roofing at 1,25 cents per square, the length of the rafter being ¾ of the breadth of the building?

Ans. $23,82.

PROBLEM IV.

Of Slaters' and Tilers' Work.

Rule—Multiply the length of the ridge by the girt from eave to eave, and in slating, allowance must be made for the double row at the bottom. In taking the girt the line is made to ply over the lowest row of slates, and returned up the under side till it meets with the wall or eaves-board; but in tiling, the line is stretched down only to the lowest part without returning it up again: double measure is generally allowed for hips, valleys, gutters, &c. but no deductions are made for chimneys.

EXAMPLES.

1. The length of a slated roof is 48 feet 6 inches, and its girt 36 feet 3 inches—What is the content?

Ans. 1708,125 square feet.

2. How many dollars will pay for tiling a barn at $3,40 per square, the length being 42 feet 6 inches, and the breadth 26 feet 9 inches, the length of the rafter being ¾ the width of the building, and the eaves projecting 9 inches on each side?

Ans. $60,148 + the expense.

PROBLEM V.

Of Plasterers' Work.

Plasterers' work is of two kinds, viz. plastering upon laths, called ceiling; and plastering upon walls, called rendering: and these different kinds must be separately measured, and their contents collected into one sum, proper deductions being made for doors and windows.

EXAMPLES.

1. If a ceiling be 64 feet 9 inches long, and 24 feet 6 inches broad, how many square yards does it contain? Ans. 32,375.

2. If the partitions between two rooms be 150 feet about, and 11 feet 6 inches high, how many square yards do they contain? Ans. 191,67.

3. If the length of a room be 21 feet 8 inches, its breadth 17 feet 4 inches, and its height 10 feet 3 inches, what will be the expense for plastering, at 7 cents per yard, deducting one door, whose size is 7 feet by 3 feet 3 inches? Ans. $8,961.

PROBLEM VI.

Of Pavers' Work.

Pavers' work is done by the square yard, and the content is found by multiplying the length by the breadth. If the dimensions be taken in feet, and the area be found in the same measure, the result being divided by 9, will give the number of square yards it contains.

EXAMPLES.

1. The length of a rectangular court-yard is 45 feet, and the breadth 18 feet 9 inches—What will be the expense of paving at 40 cents per square yard?

Ans. $37,50.

2. A rectangular court-yard is 64 feet 9 inches long, and 45 feet 6 inches in breadth—What is the expense of paving, at 45 cents per square yard?

Ans. $147,30.

3. *The paving of a square, at 40 cents per yard, cost the same that the inclosing did at 1,20 cents—What was the side of the square? Ans. 12 yards.

Of Vaulted and Arched Roofs.

Arched roofs are either vaults, domes, saloons, or groins.

Vaulted roofs are formed by arches, which spring from the opposite walls and meet in a line at the top.

Domes are formed by arches springing from a circular or polygonal base and meeting in a point at the top.

Saloons are made by arches connecting the side walls to a flat roof or ceiling in the middle groins, or are formed by the intersection of vaults with each other.

Domes and saloons rarely occur in the practice of measuring, but vaults and groins cover the cellars of most houses.

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* Divide the value of enclosing one yard square by the expense of paving one square yard, and the quotient will be the number ot yards contained in the side of the square.

Vaulted, are generally one of the three following sorts :—

1. Circular roofs, are those whose arches are some part of the circumference of a circle.

2. Elliptical roofs, are those whose arches are some part of the circumference of an ellipsis.

3. Gothic roofs, are those which are formed by circular arches meeting in a point directly over the middle of the breadth or span of the arch.

PROBLEM VII.

To find the solid content of circular, elliptical, or Gothic vaulted roofs. *Rule*—Multiply the area of one end by the length of the roof, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a semicircular vault whose span is 40 feet, and its length 124 feet?

Ans. 77911,68 the solidity required.

2. Required the solidity of an elliptic vault, whose span is 40 feet, height 12 feet, and length 80.

Ans. 30159,36 the solidity.

3. Required the solidity of a Gothic vault, whose span is 48, the chord of its arch 48, the distance of the arch from the middle of the chord 18, and the length of the vault 60 feet. Ans 136236,34512 the solidity.

PROBLEM VIII.

To find the concave or convex surface of circular elliptical, or Gothic vaulted roofs. *Rule*—Multiply the length of the arch by the length of the vault, and the product will be the superficies required.

EXAMPLES.

1. What is the concave surface of a semicircular vault, whose span is 40 feet, and its length 120?

Ans. 7539,84 feet.

2. What is the concave surface of a semicircular vault, whose span is 30 feet, and its length 90 feet?

Ans. 1413,72.

PROBLEM IX.

Given the height and dimensions of the base of a dome, to find its solidity. *Rule*—Multiply the area of the base by $\frac{2}{3}$ of the height, and the product will be the solidity required.

EXAMPLES.

1. What is the solidity of a spherical dome, the diameter of whose circular base is 60 feet?

Ans. 56548,8 feet, solidity required.

2. What is the solid content of a spherical dome, the diameter of whose circular base is 90 feet?

Ans. 763408,8 cubic feet.

PROBLEM X.

To find the superficial content of a spherical dome. *Rule*—Multiply the area of the base by 2, and the product will be the superficial content required.

EXAMPLES.

1. What is the expense of painting an octagonal spherical dome, each side of whose base is 20 feet, at 12 cents per square yard? Ans. 51,503 dollars.

2. What will be the expense of painting a hexagonal spherical dome, each side of whose base is 20 feet, at 15 cents per square yard? Ans. $34,641.

PROBLEM XI.

To find the solid content of a saloon. *Rule*—Multiply the height of the arc by its projection, and that product by $\frac{1}{4}$ of the perimeter of the ceiling, and that product again by 3,1416 together, and call the last product A.

From a side or diameter of the room, take a like side or diameter of the ceiling, and multiply the square of the remainder by the proper factor found in the table of multipliers for all regular polygons, and this product again by $\frac{2}{3}$ of the height, and call the last product B.

Multiply the area of the flat ceiling by the height of the arc, and this product added to the sum of A and B will give the content required.

EXAMPLES.

1. What is the solid content of a saloon with a circular quadrantal arc of 2 feet radius springing over a rectangular room of 20 feet long, and 16 feet wide?
Ans. 581,2629 feet.

2. Required the capacity in cubic feet of a circular building of 40 feet in diameter, and 25 feet high to the ceiling, covered with a saloon whose circular arc is 5 feet radius. Ans. 35614,8 cubic feet.

PROBLEM XII.

To find the solid content of the vacuity formed by a groin arch, either circular or elliptical. *Rule*—Multiply the area of the base by the height, and the product again by ,904, and it will give the required solidity.

EXAMPLES.

1. What is the solidity of the vacuity formed by a circular groin, each side of its square base being 12 feet? Ans. 781,056 solidity required.

2. What is the solidity of the vacuity formed by an elliptical groin, one side of its square base being 20 feet, and the height 6 feet? Ans. 2169,6 feet.

PROBLEM XIII.

To find the concave superficies of a circular groin. *Rule*—Multiply the area of the base by 1,1416, and the product will be the superficies required.

EXAMPLES.

1. What is the curve superficies of a circular groin arch, each side of its square base being 16 feet?

Ans. 292,2496 square feet.

2. What is the concave superficies of a circular groin arch, each side of its square base being 14 feet?

Ans. 223,7536 square feet.

SECTION XV.

MENSURATION OF SOLIDS.

PROBLEM I.

To compute the number of shot or shells in a finished pile.

DEFINITION.

Shot and shells are generally piled in three different forms, called triangular, square, or oblong piles, according to the form of their bases, which are either triangular, square, or rectangular.

A triangular pile is formed by the continual laying of triangular horizontal courses of shot, one above another in such a manner as that the sides of these courses called rows decrease by unity from the bottom to the top row, which ends always in one shot.

A square pile is formed by the continual laying of square horizontal courses of shot, one above another, in such manner as that the sides of these courses decrease by unity from the bottom to the top row, which ends also in one shot.

The oblong pile may be conceived as formed from the square pile to one side or face of which a number of arithmetical triangles equal to the face have been added, and the number of arithmetical triangles added to the square pile by means of which the oblong pile is formed, is always one less than the number of shot in the top row, or which is the same, equal to the difference of the bottom row of the greater side and that of the less.

To find the number of shot in a finished triangular pile. *Rule*—To the number of shot in the bottom row add 2, and multiply the sum by the number of shot in bottom row increased by one, and that product by the number of shot in the bottom row, and divide the product by 6, the quotient will be the number contained in the pile.

EXAMPLES.

1. How many shot are there in a regular triangular pile, the bottom row consisting of 8 on a side? Ans. 120.

2. How many shot are there in a finished triangular pile, containing 30 on a side in the lower tier?
Ans. 4960 shot.

3. How many shot are contained in a regular triangular pile, which has 20 in a side? Ans. 1540 shot.

PROBLEM II.

To find the number of shot contained in a finished square pile. *Rule*—Increase twice the number of shot in the side of the square by 1, and multiply the sum by the number increased by 1, and that product by the number, and divide the product by 6, the quotient will be the number required.

EXAMPLES.

1. How many shot or shells are there in a finished square pile, containing 20 in a side? Ans. 2870.

2. What is the number of shot contained in a finished square pile, the lower tier containing 30 in each side?
Ans. 9455.

3. Required the number of shot in a finished square pile, containing 12 in each side? Ans. 650.

PROBLEM III.

To find the number of shot or shells contained in a finished oblong pile, the number of courses and the number in the top row being given. *Rule*—To twice the number of courses increased by 1, add the product of the number less by 1, in the top row multiplied by 3, and multiply that sum by the product of the number of courses increased by 1, and that product again by the number of courses; divide the last product by 6, and the quotient will be the number sought.

EXAMPLES.

1. The number of courses in a finished oblong pile of shells is 30, and the number in the top row is 31, required the number contained in the pile? Ans. 23405 shells.

2. The number of courses of a finished oblong pile is 20, and the number in the upper course is 24, required the number contained in said pile? Ans. 7700.

3. The number of shot in the upper course of a finished oblong pile is 41, and the number of courses 30, how many shot are contained in said pile? Ans. 28055 shot.

PROBLEM IV.

To find the number of shot contained in an incomplete pile of either of the aforementioned forms. *Rule*—Find the number of shot which would be contained in the pile if it were complete, according to the rule for its proper form, and likewise the number it would take to complete the same, subtract one result from the other and the remainder will be the number contained in the unfinished pile.

EXAMPLES.

1. What is the number of shot contained in the incomplete triangular pile, one side of the bottom course being 40 and the top course 20? Ans. 10150.

2. How many shot are contained in the incomplete triangular pile, the side of the base containing 24 and the top 8? Ans. 2516.

3. How many cannonballs are contained in the unfinished square pile, the side of the base containing 24 and the top 8? Ans. 4760.

4. What is the number of cannon shot contained in the unfinished square pile, the base having 30 on a side and the top 12? Ans. 8949.

5. How many shot are in the unfinished rectangular pile of 12 courses, the length of the base being 40 and 20? Ans. 6144.

6. Required the number of shot contained in the unfinished rectangular pile of 8 courses, the length and breadth of the base containing 30 and 20 shot.
Ans. 3456.

SECTION XVI.

OF SPECIFIC GRAVITY.

THE Specific Gravities of bodies are their relative weights contained under the same given magnitude as a cubic foot, a cubic inch, &c.

The specific gravities of several sorts of bodies are expressed by the numbers annexed to their names in the following table, as found by actual experiments, and are calculated to correspond with a cubic foot of each in avoirdupois ounces.

A TABLE OF SPECIFIC GRAVITIES OF BODIES.

Platina (pure) one cubic foot weighs	23000 ounces.
Fine gold - - - -	19400
Standard gold - - -	17724
Quick silver (pure) - - -	14000
Quick silver (common) - -	13600
Lead - - - - -	11325
Fine silver - - - -	11091
Standard silver - - -	10535
Copper - - - - -	9000
Gun metal - - - -	8784
Cast brass - - - -	8000
Steel - - - - -	7850
Iron - - - - -	7645
Cast Iron - - - -	7425
Tin - - - - -	7320
Clear crystal glass - - -	3150
Granite - - - -	3000
Marble and hard stone -	2700
Common green glass - -	2600
Flint - - - - -	2570
Common stone - - -	2520

Clay and loam - - -	2160 ounces.
Brick - - - - -	2000
Common earth - - -	1984
Nitre - - - - -	1900
Ivory - - - - -	1825
Brimstone - - - -	1810
Solid gunpowder - - -	1745
Sand - - - - -	1520
Coal - - - - -	1250
Pitch - - - - -	1150
Dry boxwood - - - -	1030
Sea water - - - -	1030
Common water - - -	1000
Dry oak - - - -	925
Gunpowder closely shaken -	922
Dry ash - - - - -	800
Dry maple - - - -	755
Dry elm - - - -	600
Dry fur - - - - -	550
Cork - - - - -	240
Air at a mean state - - -	$1\frac{2}{9}$

PROBLEM I.

To find the specific gravity of a body. *Rule*—When the body is heavier than water weigh it both in water and out of water, and take the difference which will be the weight lost in water. Then, as the weight lost in water is to the whole or absolute weight, so is the specific gravity of water to the specific gravity of the body.

EXAMPLES.

1. If a stone weigh 10*lb*., but in water only 7, required its specific gravity. Ans. 3333 + avoirdupois ounces.

2. If a stone weigh 24*lbs*., but in water only 20, what is its specific gravity. Ans. 6000.

PROBLEM II.

When the body is lighter than water so that it will not sink, affix to it some other body heavier than water, so that the mass compounded of the two may sink together. Weigh the heavier body and the compound mass separately, both in water and out of it, and find how much each loses in water, by subtracting its weight in water from its weight in air. Then as the difference of these remainders is to the weight of the light body in air, so is the specific gravity of water to the specific gravity of the body.

EXAMPLES.

1. Suppose a piece of elm weighs 15*lb.* in the air, and that a piece of copper which weighs 18*lb.* in air and 16*lb.* in water, is affixed to it, and that the compound weighs 6*lb.* in water—Required the specific gravity of the elm. Ans. 600.

2. Suppose a piece of dry oak weighs in air 37*lb.*, and a piece of copper that weighs 18*lb.* in air and 16 in water, is affixed to it, and that the compound weighs 13*lb.* in water—Required the specific gravity of the oak. Ans. 925 ounces.

PROBLEM III.

To find the specific gravity of a fluid. *Rule*—Take a piece of a body of known specific gravity, weigh it both in and out of the fluid, finding the loss of weight by taking the difference of the two; then as the absolute weight is to the loss of weight, so is the specific gravity of the solid to the specific gravity of the fluid.

EXAMPLES.

1. A piece of cast iron weighed 34,61 ounces in a fluid and 40 ounces out of it—What is the specific gravity of the fluid? Ans. 1000.

2. A piece of cast brass weighed 35,375 ounces in a fluid, and out of it 40 ounces—What was the specific gravity of the fluid? Ans. 925.

PROBLEM IV.

To find the quantities of two ingredients in a given compound. *Rule*—Take the difference of every pair of the three specific gravities, viz. of the compound and each ingredient, and multiply the difference of every two by the third; then as the greatest product is to the whole weight of the compound, so is each of the other products to the weights of the two ingredients.

EXAMPLES.

1. A composition of 112*lb.*, being made of tin and copper, whose specific gravity is found to be 8784—Required the quantity of each. Ans. 100 cop.–120 tin.

2. A composition of 48*lb.*, being made of tin and copper, whose specific gravity is found to be 8440—What was the weight of each ingredient?

Ans. 34,1225 of copper, and 13,8775 of tin.

PROBLEM V.

The weight of a body being given to find its magnitude. *Rule*—As the tabular specific gravity of the body is to its weight in avoirdupois ounces, so is one cubic foot, or 1728 cubic inches to its solidity, in feet or inches respectively.

EXAMPLES.

1. Required the content of an irregular block of marble that weighs 112 pounds.

Ans. 71,68 cubic inches.

2. Required the solidity of an irregular block of granite that weighs 300 pounds.

Ans. 172,8 cubic inches.

PROBLEM VI.

To find the weight of a body when its magnitude is given. *Rule*—As one cubic foot is to the content of the body, so is its tabular specific gravity to the weight of the body.

EXAMPLES.

1. Required the weight of a bar of iron 2½ inches broad, ½ inch in thickness, and 16 feet long.

Ans. 66,3625 pounds.

2. Required the weight of a block of marble, whose length is 63 feet, and its breadth and thickness each 12 feet, these being the dimensions of one of the stones in the walls of Balbec.

Ans. $683\frac{7}{16}$ tons, which is equal to the burden of an East India ship.

3. What is the weight of a pint, ale measure, of gunpowder closely shaken? Ans. 18,8 ounces.

4. What is the weight of a cord of dry maple wood? Ans. 5400 nearly.

5. What is the weight of a block of dry oak, which measures 12 feet long, 16 inches wide, and 14 inches thick? Ans. 1079 pounds.

6. A common stone is 4 feet long, 2 feet 6 inches wide, and one foot 9 inches thick—What is its weight? Ans. 2756,25 pounds.

7. What is the weight of a leaden ball $\frac{3}{4}$ of an inch diameter? Ans. 1,4477 — ounces.

8. What is the weight of a cubic inch of standard gold? Ans. 10,257 — avoirdupois ounces.

SECTION XVII.

MISCELLANEOUS QUESTIONS.

1. How many square feet in a floor 16 feet 9 inches square? Ans. 280,5625 square feet.

2. If the floor of a room be 24 feet long, and 16 feet 9 inches wide, what is the expense of flooring at $1,50 per square? Ans. $6,03.

3. If a piece of land be 40 chains in length, and 35 in breadth, how many acres does it contain? Ans. 140 acres.

4. The base and perpendicular of a right-angled triangle are 36 and 24 chains—What is the area and length of the hypotenuse?

Ans. 31,2 acres, 43,2666 + chains the length of the hypotenuse.

5. The area of a rectangular parallelogram is 4,5 acres, and the length exceeds the breadth by four chains—What is the length and breadth of the parallelogram? Ans. 9 chains by 5.

6. How much in length, that is 8 inches wide, will be equal to a square foot? Ans. 18 inches.

7. The diameter of a circle is 25 rods—Required the length of a stone wall that will enclose it. Ans. 78,54 rods.

8. What is the length of a stone wall that will enclose an acre of land in the form of a circle? Ans. 44,84 rods.

9. How many perches of stone are sufficient to build a stone wall two feet thick, four feet six inches high, enclosing a circular garden, containing half an acre of land? Ans. 288,3 perches.

10. How many men may stand on an acre of land, allowing 12 square feet to each? Ans. 3630 men.

11. The transverse and conjugate diameters of an ellipsis, are 46 and 36 rods—What is its area? Ans. 8,1289 acres.

12. Two men carrying a weight of 128 pounds on a pole four feet long, the weight hanging 6 inches from the centre—How much of its weight does one carry more than the other? Ans. 32 pounds.

13. The diameter of a globe is 18 inches—What is its solidity? Ans. 1,76715 cubic feet.

14. I demand the length of the chord that will cut off $\frac{1}{3}$ of the area of a circle, whose diameter is 60 chains. Ans. 57,856 chains.

15. The versed sine of the segment of a circle is 16 feet, and the chord 64—What is the diameter of the circle from which the segment was taken? Ans. 80 feet.

16 The area of a rectangular parallelogram is 10 acres, and the length of the sides are in the proportion of five to four—What are the length of the sides? Ans. 11,18 +, and 8,944 + chains.

17. What is the length of a line which will allow my gardener to strike out a round orangery containing 40 rods of ground? Ans. 19,625 yards.

18. What is the solidity of a cylinder whose length is 42 feet, and diameter 17 inches? Ans. 66,2 +.

19. What length of wire, without loss of metal, may be drawn from a globe 6 inches diameter, the diameter of the wire being $\frac{1}{20}$ part of an inch? Ans. 1600 yards.

20. What is the solidity of a cast iron stove that weighs 400 pounds? Ans. 1489,45 + cubic inches.

21. What is the solidity of a cone, whose diameter at the base is 40 inches, and perpendicular height 9 feet? Ans. 26,18 cubic feet.

22. How many solid yards of earth must be thrown out to make a cellar 48 feet long, 27 wide, and 6 feet deep? Ans. 288 solid yards.

23. How many bushels will a cubic box, whose side is 4 feet 6 inches, contain? Ans. 73,2245 + bushels.

24. The perpendicular altitude of a cone is 8 feet—

What is the altitude when ⅛ of its solidity is cut off by a section parallel to, and adjoining its base?

Ans. 7,65 feet.

25. The base of a right-angled triangle is 50 chains, and its perpendicular 40, to be divided into two equal parts by a line parallel to the perpendicular—Required the length of the base of each part?

Ans. 35,3553 + and 14,6447.

Take again

26. The area of a rectangular parallelogram is ten acres, 3 roods, and 8 rods, and the length of the diagonal is 60 rods—What is the length of the sides?

Ans. 48 and 36 rods.

27. How many pieces ¼ of an inch square are contained in a circular piece of plank 6 inches diameter, and 1½ inches thick? Ans. 2714 + pieces.

28. How many ale gallons are contained in a cistern whose top diameter is 5 feet, bottom 6, and depth 7 feet.

Ans. 1021,8889 gallons.

29. A well is dug 30 feet deep, 7 feet in diameter, and stoned so that the hollow space is 2 feet 6 inches diameter—How many perches of stone were required to complete the same? Ans. 61,047 perches.

30. Required the transverse and conjugate diameters of an ellipsis in the proportion of 4 to 3, that will contain one acre of ground.

Ans. 16,43 — rods the transverse, and 12,32 +, the conjugate required.

31. What is the diameter of a solid globe of glass, which when blown into a hollow globe until the shell is ¼ of an inch in thickness, will be sufficient to contain ten gallons of wine? Ans. 7,45 + inches.

32. What must be the depth of a cistern, whose diameters are 5 feet, and 6, to contain 30 barrels?

Ans. 5,3 feet.

33. Suppose a tree 100 feet long should break off, so that the top should reach the ground 40 feet from the base, the broken part resting upon the stump or upright part—What would be the length of the broken piece.

Ans. 58 feet.

34. The mountain Teneriffe, is 2,5 miles high—How far may it be seen by an eye elevated five feet above the surface, allowing the diameter of the earth to be 8000 miles? Ans. 144,195 + miles.

35. What is the diameter of that globe, whose solidity is equal to $\frac{2}{3}$ of the area of its surface? Ans. 4 inches.

36. How many cannon balls, 4 inches diameter, without loss of metal, may be run from a ton of cast iron?
Ans. 249.

37. If the three sides of a scalene triangle be 40, 50, and 60 rods, how many acres does it contain?
Ans. 6,2 acres.

38. The difference between the diagonal of a square and one of its sides is 6 chains—What is its area?
Ans. 21,025 — acres.

39. The area of a scalene triangle is 9,6 acres, the base is 20 chains, and the other two sides are in the proportion of 3 to 4—What is the length of those sides?
Ans. 16 and 12 chains.

40. The two diameters of an oblate spheroid are in the proportion of 5 to 6, and its solidity is equal to that of a globe 18 inches diameter—What are the two diameters of the spheroid?
Ans. 1,918 — and 1,598 + feet.

41. What is the difference in solidity between an oblate spheroid, whose diameters are 18 and 15 inches, and a prolate spheroid of the same dimensions?
Ans. 424,116 cubic inches.

42. If a cistern whose diameters are 48 and 60 inches, and depth 6 feet, be filled half full of water, what will be the depth? Ans. 2,676 — feet.

43. The area of an equilateral triangle is 24 acres—What is the length of its sides?
Ans. 23,5426 + chains.

44. What is the length of the sides of a cubic box that contains 60 bushels? Ans. 50,53 + inches.

45. A circular garden contains half an acre, which is surrounded by a gravel walk that contains $\frac{1}{4}$ of the area of the garden—What is the width of the walk?
Ans. ,595 rods.

46. How many perches of stone will be sufficient to make a cellar wall under a building 42 feet long, and 26 feet wide, provided the wall be two feet thick, and 7 feet high? Ans. 108,6 + perches.

47. What is the solidity of an iron anchor that weighs 2 tons? Ans. 9,376 + cubic feet.

48. Required the solidity of the largest equilateral triangular pyramid, which may be cut from a globe 3 feet in diameter, and likewise the solidity of the chips that reduces it to a pyramid.

Ans. 2,2083 + cubic feet the solidity of the pyramid, and 11,9289 the solidity of the chips.

49. Suppose the diameter of a circle is 60 chains—What is the length of the arc contained between the two radii, including an angle of 24 degrees?

Ans. 12,5664.

50. The area of a rectangular parallelogram is 5 acres, and the wall that encloses it is 120 rods in length—What is the length of its sides? Ans. 40 and 20 rods.

51. The length of the side of an octagon is 20 feet—How many square yards of ground does it stand on?

Ans. 214,5967 + square yards.

52. The depth, top, and bottom diameters of a copper kettle, are respectively in the proportion of 6, 5, and 4, and it contains 25 ale gallons—What is the depth, and likewise the diameters of the bottom and top?

Ans. depth 25,14 inches, top diameter 20,95, and the bottom 16,76 inches.

53. What must be the diameter of a cylinder 60 feet long, to make 90 solid feet when hewn square?

Ans. 1,732 feet.

54. The three sides of a scalene triangle containing 60 acres, are in the proportion of 5, 8, and 7—What is the length of the sides?

Ans. 47,0848, 41,1992, and 29,428 chains.

55. The sides of a scalene triangle, are 40, 50, and 60 chains, to be divided into three equal parts by lines parallel to the base—Required the length of the base of each part? Ans. 49 — and 34,636 — chains.

56. The two equal sides of an isosceles triangle are each 30 chains, and the third side 20 chains, to be divided into two parts, in the proportion of three to two, the greater part parallel to, and next the base—Required the length of the sides of the smaller part.

Ans. 19—chains the length of each of the equal sides, and 12,65 — the base.

57. *The transverse and conjugate diameter of an ellipsis are 60 and 40 chains—What is its periphery?

Ans. 157,555 chains.

58. The base of a right-angled triangle is 40 chains, and the perpendicular 30—What is the diameter of the greatest circle that can be inscribed within it?

Ans. 20 chains.

59. The hypotenuse of a right-angled triangle is 42 chains; and if a perpendicular be erected from the end of the hypotenuse to intercept a line drawn from the point of the triangle, the length of the perpendicular and that line will be 124 chains—What is the length of each side separately?

Ans. 69,1129 chains, and 54,8871 —.

60. The chord of the segment of a circle is 11 inches, and the diameter of the circle from which it was taken 18 inches—What is the versed sine?

Ans. 1,8771 inches.

61. The length of three perpendiculars dropt from a point within an equilateral triangle is 27 chains—What is the area of the triangle? Ans. 10,52429 + acres.

62. What is the diameter of a globe whose greatest inscribed cube is 16 inches on a side?

Ans. 27,7 + inches.

63. What length of a cylinder, 16 inches diameter, will make 36 solid feet, when hewn square?

Ans. 40,5 feet.

64. What is the length of the side of a square piece

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* Multiply the sum of the squares of the transverse and conjugate diameters by four, and extract the square root of the product, to which root add ⅙ of the conjugate, the sum will be the curve of the ellipsis required.

of land which contains as many acres as there are chains in the perimeter? Ans. 40 chains.

65. The difference between a circumscribing square and the inscribed circle is four acres—How many chains are contained in the length of the side?

Ans. 13,666 +.

66. The perpendicular altitude of a square pyramid is 18 feet, which is to be divided by sections parallel to the base, into three equal parts—Required the perpendicular altitude of each part?

Ans. 12,48, 3,244, and 2,276 feet.

67. The sum of the base and perpendicular of a right-angled triangle is 70 rods, and the hypotenuse is 50—What is the separate length of the sides?

Ans. 40 and 30 rods.

68. What are the dimensions of a cistern sufficient to hold 30 barrels, whose length, top, and bottom diameters are in the proportion of 5, 4, and 3.

Ans. 6,883 + feet the length, 5,5064 feet bottom diameter, 4,1298 top diameter.

69. *The separate lengths of three lines drawn from a point within an equilateral triangle, to each of the angles are 20, 30, and 45 chains—Required the area of the triangle. Ans. 116,9066 + acres.

70. What is the length of the side of a cubic box which contains its largest inscribed globe, and 1 bushel to fill the vacancies? Ans. 16,526 + inches.

71. The sum of the diagonal, and breadth of a parallelogram is 64 chains, and the length 40—What is the area? Ans. 78 acres.

72. The foot of a ladder 40 feet long, was so placed, that at one side of the street it reached the building 24 feet from the ground, and on the opposite 16 feet—What was the breadth of the street?

Ans. 68 feet 8 inches.

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* Find the mean proportion between the greater and less of the given numbers and multiply it by two, and to that product add the other given distance, and divide the sum by the square root of the number three, the quotient will be the length of the side nearly.

73. The side of a rhombus is 30 chains, and the perpendicular 12—What is the length of the longer diagonal?
Ans. 58,739 —.

74. The longer diagonal of a rhombus is 24 chains, and the shorter 18—What is the length of the sides?
Ans. 15 chains.

75. The transverse and conjugate diameters of an ellipsis are 42 and 36 rods—What is the length of the side of an equilateral triangle that contains half the area?
Ans. 37,03 rods.

76. The distance of the centres of two circles, whose diameters are each 50 rods, is equal to 30 rods—How many acres are enclosed by their circumferences?
Ans. 3368,285 square rods = 21,0518 — acres.

77. A gentleman has a garden 100 feet long, and 60 feet wide, and a gravel walk is to be made of an equal width half round it—What must the width of the walk be, so as to take up just $\frac{1}{4}$ of the ground? Ans. 10 feet.

78. What is the expense of painting a conical spire whose altitude is 120 feet, and circumference at the base 60 feet, at 8 cents per yard?
Ans. \$3,21.

79. A cubical foot of brass is to be drawn into a wire of $\frac{1}{40}$ of an inch in diameter—What will be the length of the wire, allowing no loss in the metal?
Ans. 5,556 + miles.

80. Three men bought a tapering piece of timber which was the frustrum of a square pyramid; one side of the greater end was 3 feet, one side of the less end 1 foot, and the length 18 feet—What is the length of each man's part, supposing they paid equally, and consequently have equal shares?
Ans. 10,17, 4,56, and 3,27 feet.

81. The solidity of a bushel measure is 2150,4252 solid inches—What is the diameter of a circular half bushel whose depth is 8 inches?
Ans. 12,452 inches the diameter.

82. If a conical glass whose diameter is 5 inches, and altitude 6 inches, be filled full of water, and a globe 4 inches diameter be dropt in the glass, so that from

the centre of the ball to the vertex of the glass shall be 5,148 inches, how many cubic inches of water will be displaced by the ball? Ans. 26,8144 cubic inches.

83. Suppose two wheels, the one 5 feet diameter, and the other 4, be placed on an axletree 12 feet long, and set rolling on a plane till they describe a circle—What is the diameter of the circle made by the greater wheel, and the area of the ring contained between the circles.

Ans. 120,1 + feet the diameter, and 4075,28352 square feet the area of the ring.

84. Beginning at the northeast corner and running south 10 chains, thence west 20, thence north 8 chains, thence to the place of beginning; it is required to lay off from the southwest corner on the south line, 8 acres—How far from the southwest corner must the line be drawn, and what will be its length?

Ans. 8,94427 + chains its length, and 9,4427 chains from the southwest corner.

85. How far from the vertex of a cone measured on the slope, whose diameter at the base is 4 feet, and perpendicular altitude 12 feet, must it be cut off so that $\frac{3}{4}$ of the timber may be left at the vertex?

Ans. 11,05 feet.

86. If the diameter of the earth at the equator be 8000 miles, what is the diameter at the 45th degree of north latitude, allowing the earth to be a complete sphere? Ans. 5656,85424 miles.

87. Being on the side of a river, and wishing to know its breadth, and having no instrument for measuring distances excepting a rule two feet in length, I set a stake where I stood, and measured back from the river 20 feet, and set a stake directly to range with the former and a tree which grew on the opposite bank; then at right angles from this place I measured the distance of 16 feet, and set another stake, then on the line first drawn at the river's brink I set another stake directly between the tree and the stake; and lastly, measured the distance between the stakes at the river's brink, and

found them to be 15 feet 6 inches asunder—What was then the breadth of the stream? Ans. 620 feet.

88. From a point within an equilateral triangle I measured the distances to each of the angles, and found that if $\frac{1}{3}$ of the two greater distances were added to the less, the sum would be 25 chains; and if $\frac{1}{4}$ of the less and greater were added to the other, the sum would be the same; and if $\frac{1}{5}$ of the two less were added to the greater, the sum would likewise be 25 chains—what was the length of the sides of the equilateral triangle, and the distances from each of the angles to the point within.

Ans. 13, 17, and 19 chains the distances, and 27,96 + chains the length of the side.

89. Four men bought a grindstone of 5 feet in diameter—How much of its diameter must each grind off to have equal shares of the stone, if one first grind his part, and then the next, &c. till the stone is ground away, making no allowance for the eye?

Ans. 8, 9,5736, 12,4264, and 30 inches.

90. The area of a certain piece of land contained between three equal circles whose peripheries touched each other, is two acres—What is the diameter of that circle which will just enclose the three?

Ans. 47,98646.

91. Three gentlemen's seats are in the form of a scalene triangle; from A to B is 2,5 miles, from A to C is 2,4, and from B to C 2 miles—What will be the distance to a school house equally distant from each of the seats? Ans. 1,343 miles.

92. The three sides of a scalene triangle are 20, 30, and 40 rods—What is the diameter of a circle which will pass through all three of the angles?

Ans. 41,312 + rods.

93. *The longest diagonal of a rhombus is 21 chains, and the area is 48 square chains, and the longest and

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* To the square of half the diagonal add the perpendicular, and likewise its square, and multiply the sum by 4, the square root of the product will be the sum of the required sides; you will then have the sum of two sides and their proportion to find the sides separately.

shortest sides are in the proportion of 85 to 50—What are the lengths of the sides? Ans. 17 and 10 chains.

94. A gentleman purchased a certain tract of land in the form of a circle, at 20 dollars per acre, the dollars which encompassed it exactly paid for it. If a dollar be an inch and a half in diameter, what was the diameter of the piece of land, and what did it cost?

Ans. 1056 chains the diameter, and it cost 1751655,6288 dollars.

95. The side of a hexagon is 16 feet—What is its area, and likewise the length of a perpendicular let fall from the centre to the middle of one of its sides?

Ans. 665,1074 + square feet the area, and 13,8564 feet the perpendicular.

96. The area of a regular pentagonal field is 5 acres, 2 roods, and 25 perches—Required the length of the side. Ans. 22,935 + perches.

97. Suppose the diameter of the earth to be 8000 miles, and the moon 2180, and that at a mean rate they are 240000 miles asunder—How far from the earth is the centre of gravity, provided their densities be equal?

Ans. 856,35875 miles above the surface.

98. A father bequeathed to his three sons the greatest equal circles that could be formed within the periphery of one whose diameter was 100 chains, and the remainder to the widow—How many acres was the portion of each?

Ans. 169,167716 acres each of the son's share, and 277,896852 acres the widow's part.

99. The base of a scalene triangle is 40 chains, and the other two sides are in the proportion of 3 to 2, and if a right line be drawn from the middle of the base to its opposite angle, its square will be 250—Required the length of the other two sides.

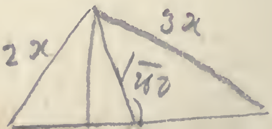
Ans. 30 and 20 chains.

100. A gentleman owning a farm containing 785,4 acres in the form of a circle, gave to his nine daughters the nine largest pentagons that could be formed with an angle of each touching the periphery of the circle;

$400 - 40m + m^2$

$4x^2 - \cancel{y^2} = 9x^2 - 1600 + 8m - \cancel{y^2}$

$1600 - 8m = 5x^2$



to his son the largest nonagon that could be formed about the bases of the pentagons; and the remainder to the widow—What was the share of each?

Ans. 191,53578 acres the area of the nonagon, and consequently the son's share; 53,30674 acres each of the daughters' share; and 114,10356 acres the widow's part.

A
TABLE
OF THE
AREAS OF THE SEGMENTS OF A CIRCLE WHOSE DIAMETER IS UNITY, AND SUPPOSED TO BE DIVIDED INTO ONE THOUSAND EQUAL PARTS.

Versed sine.	Segments. area.	Versed sine.	Segments. area.	Versed sine.	Segments. area.
,001	,000042	,024	,004921	,047	,013392
,002	,000119	,025	,005230	,048	,013818
,003	,000219	,025	,005546	,049	,014247
,004	,000337	,027	,005867	,050	,014681
,005	,000470	,028	,006194	,051	,015119
,006	,000618	,029	,006527	,052	,015561
,007	,000779	,030	,006865	,053	,016007
,008	,000951	,031	,007209	,054	,016457
,009	,001135	,032	,007558	,055	,016911
,010	,001329	,033	,007913	,056	,017369
,011	,001533	,034	,008273	,057	,017831
,012	,001746	,035	,008638	,058	,018296
,013	,001968	,036	,009008	,059	,018766
,014	,002199	,037	,009383	,060	,019239
,015	,002438	,038	,009763	,061	,019716
,016	,002685	,039	,010140	,062	,020196
,017	,002940	,040	,010537	,063	,020680
,018	,003202	,041	,010931	,064	,021168
,019	,003471	,042	,011330	,065	,021659
,020	,003748	,043	,011734	,066	,022154
,021	,004031	,044	,012142	,067	,022652
,022	,004322	,045	,012554	,068	,023154
,022	,004618	,046	,012971	,069	,023659

Versed sines.	Segments. area.	Versed sines.	Segments. area.	Versed sines.	Segments. area.
,070	,024168	,102	,042080	,134	,062707
,071	,024680	,103	,042687	,135	,063389
,072	,025195	,104	,043296	,136	,064074
,073	,025714	,105	,043908	,137	,064760
,074	,026236	,106	,044522	,138	,065449
,075	,026761	,107	,045139	,139	,066140
,076	,027289	,108	,045759	,140	,066833
,077	,027821	,109	,046381	,141	,067528
,078	,028356	,110	,047005	,142	,068225
,079	,028894	,111	,047632	,143	,068924
,080	,029435	,112	,048262	,144	,069625
,081	,029979	,113	,048894	,145	,070328
,082	,030526	,114	,049528	,146	,071033
,083	,031076	,115	,050165	,147	,071741
,084	,031629	,116	,050804	,148	,072450
,085	,032186	,117	,051446	,149	,073161
,086	,032745	,118	,052090	,150	,073874
,087	,033307	,119	,052736	,151	,074589
,088	,033872	,120	,053385	,152	,075306
,089	,034441	,121	,054036	,153	,076026
,090	,035011	,122	,054689	,154	,076747
,091	,035585	,123	,055345	,155	,077469
,092	,036162	,124	,056003	,156	,078194
,093	,036741	,125	,056663	,157	,078921
,094	,037323	,126	,057326	,158	,079649
,095	,037909	,127	,057991	,159	,080380
,096	,038496	,128	,058658	,160	,081112
,097	,039087	,129	,059327	,161	,081846
,098	,039680	,130	,059999	,162	,082582
,099	,040276	,131	,060672	,163	,083320
,100	,040875	,132	,061348	,164	,084059
,101	,041476	,133	,062026	,165	,084801

Versed sines.	Segments. area.	Versed sines.	Segments. area.	Versed sines.	Segments. area.
,166	,085544	,198	,110226	,230	,136465
,167	,086289	,199	,111024	,231	,137307
,168	,087036	,200	,111823	,232	,138150
,169	,087785	,201	,112624	,233	,138995
,170	,088535	,202	,113426	,234	,139841
,171	,089287	,203	,114230	,235	,140688
,172	,090041	,204	,115035	,236	,141537
,173	,090797	,205	,115842	,237	,142387
,174	,091554	,206	,116650	,238	,143238
,175	,092313	,207	,117460	,239	,144091
,176	,093074	,208	,118271	,240	,144944
,177	,093836	,209	,119083	,241	,145799
,178	,094601	,210	,119897	,242	,146655
,179	,095366	,211	,120712	,243	,147512
,180	,096134	,212	,121529	,244	,148371
,181	,096903	,213	,122347	,245	,149230
,182	,097674	,214	,123167	,246	,150091
,183	,098447	,215	,123988	,247	,150953
,184	,099221	,216	,124810	,248	,151816
,185	,099997	,217	,125634	,249	,152680
,186	,100774	,218	,126459	,250	,153546
,187	,101553	,219	,127285	,251	,154412
,188	,102334	,220	,128113	,252	,155280
,189	,103116	,221	,128942	,253	,156149
,190	,103900	,222	,129773	,254	,157019
,191	,104685	,223	,130605	,255	,157890
,192	,105472	,224	,131438	,256	,158762
,193	,106261	,225	,132272	,257	,159636
,194	,107051	,226	,133108	,258	,160510
,195	,107842	,227	,133945	,259	,161386
,196	,108636	,228	,134784	,260	,162263
,197	,109430	,229	,135624	,261	,163140

Versed sines.	Segments. area.	Versed sines.	Segments. area.	Versed sines.	Segmenta area.
,262	,164019	,294	,192684	,326	,222277
,263	,164899	,295	,193596	,327	,223215
,264	,165780	,296	,194509	,328	,224154
,265	,166663	,297	,195422	,329	,225093
,266	,167546	,298	,196337	,330	,226033
,267	,168430	,299	,197252	,331	,226974
,268	,169315	,300	,198168	,332	,227915
,269	,170202	,301	,199085	,333	,228858
,270	,171089	,302	,200003	,334	,229801
,271	,171978	,303	,200922	,335	,230745
,272	,172867	,304	,201841	,336	,231689
,273	,173758	,305	,202761	,337	,232634
,274	,174649	,306	,203683	,338	,233580
,275	,175542	,307	,204605	,339	,234526
,276	,176435	,308	,205527	,340	,235473
,277	,177330	,309	,206451	,341	,236421
,278	,178225	,310	,207376	,342	,237369
,279	,179122	,311	,208301	,343	,238318
,280	,180019	,312	,209227	,344	,239268
,281	,180918	,313	,210154	,345	,240218
,282	,181817	,314	,211082	,346	,241169
,283	,182718	,315	,212011	,347	,242121
,284	,183619	,316	,212940	,348	,243074
,285	,184521	,317	,213871	,349	,244026
,286	,185425	,318	,214802	,350	,244980
,287	,186329	,319	,215733	,351	,245934
,288	,187234	,320	,216666	,352	,246889
,289	,188140	,321	,217599	,353	,247845
,290	,189047	,322	,218533	,354	,248801
,291	,189955	,323	,219468	,355	,249757
,292	,190864	,324	,220404	,356	,250715
,293	,191775	,325	,221340	,357	,251673

Versed sines.	Segments. area.	Versed sines.	Segments. area.	Versed sines.	Segments. area.
,358	,252631	,390	,283592	,422	,315016
,359	,253590	,391	,284568	,423	,316004
,360	,254550	,392	,285544	,424	,316992
,361	,255510	,393	,286521	,425	,317981
,362	,256471	,394	,287498	,426	,318970
,363	,257433	,395	,288476	,427	,319959
,364	,258395	,396	,289453	,428	,320948
,365	,259357	,397	,290432	,429	,321938
,366	,260320	,398	,291411	,430	,322928
,367	,261284	,399	,292390	,431	,323918
,368	,262248	,400	,293369	,432	,324909
,369	,263213	,401	,294349	,433	,325900
,370	,264178	,402	,295330	,434	,326892
,371	,265144	,403	,296311	,435	,327882
,372	,266111	,404	,297292	,436	,328874
,373	,267078	,405	,298273	,437	,329866
,374	,268045	,406	,299255	,438	,330858
,375	,269013	,407	,300238	,439	,331850
,376	,269982	,408	,301220	,440	,332843
,377	,270951	,409	,302203	,441	,333836
,378	,271920	,410	,303187	,442	,334829
,379	,272890	,411	,304171	,443	,335822
,380	,273861	,412	,305155	,444	,336816
,381	,274832	,413	,306140	,445	,337810
,382	,275803	,414	,307125	,446	,338804
,383	,276775	,415	,308110	,447	,339798
,384	,277748	,416	,309095	,448	,340793
,385	,278721	,417	,310081	,449	,341787
,386	,279694	,418	,311068	,450	,342782
,387	,280668	,419	,312054	,451	,343777
,288	,281642	,420	,313041	,452	,344772
,389	,282617	,421	,314029	,453	,345768

Versed sines.	Segments. area.	Versed sines.	Segments. area.	Versed sines.	Segments. area.
,454	,346764	,470	,362717	,486	,378701
,455	,347759	,471	,363715	,487	,379700
,456	,348755	,472	,364713	,488	,380700
,457	,349752	,473	,365712	,489	,381699
,458	,350748	,474	,366710	,490	,382699
,459	,351745	,475	,367709	,491	,383699
,460	,352741	,476	,368708	,492	,384699
,461	,353739	,477	,369707	,493	,385699
,462	,354736	,478	,370706	,494	,386699
,463	,355732	,479	,371705	,495	,387699
,464	,356730	,480	,372674	,496	,388699
,465	,357727	,481	,373703	,497	,389699
,466	,358725	,482	,374702	,498	,390699
,467	,359723	,483	,375702	,499	,391699
,468	,360721	,484	,376702	,500	,392699
,469	,361719	,485	,377701		

THE END.

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